L12 - Generative Design

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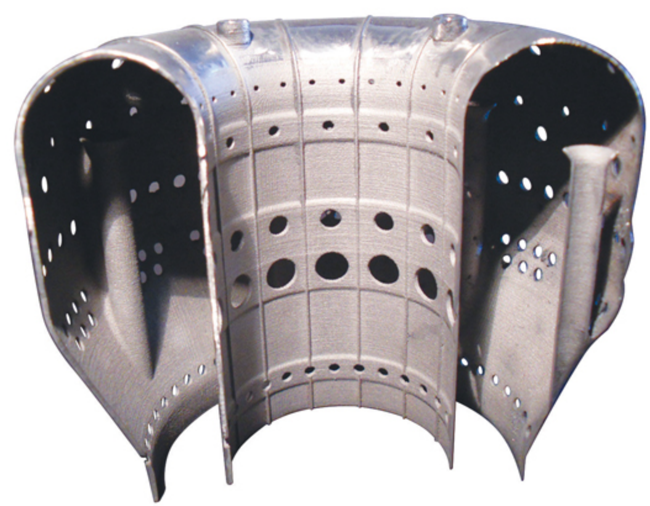
November 24, 2018

Additive Manufacturing

- Defined by ASTM as:
  - Process of joining materials to make objects from 3D model data, usually layer upon layer
- Six Different Types of AM:
  - Lasers: Stereolithography Apparatus (SLA), Selective Laser Sintering (SLS)
  - Nozzles: Fused Deposition Modeling (FDM)
  - Print-heads: Multi-jet Modeling (MJM), Binder-jet Printing (3DP)
  - Cutters: Laminated Object Modeling (LOM)
- Mainly used for Rapid Prototyping (Past)
- More and More used for 'Mass'-Production (Present)

Example of Printed Consumer Products

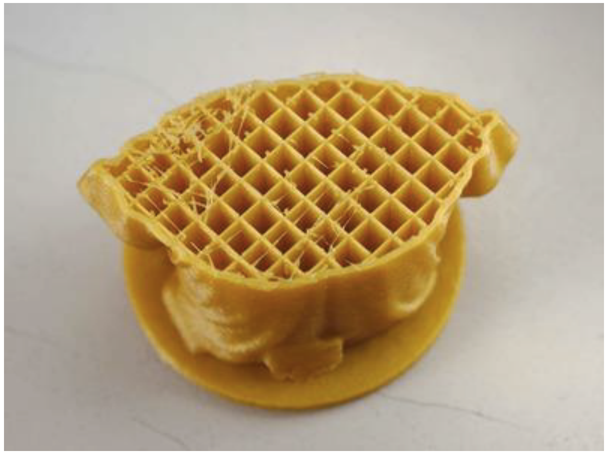
Boeing Air-Ducting Parts



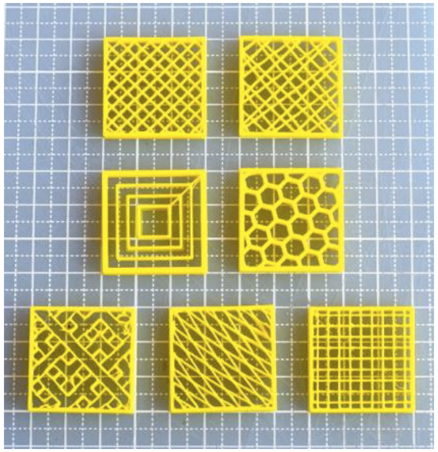
Air-Intake for a Turbine

Lightweight Parts by AM

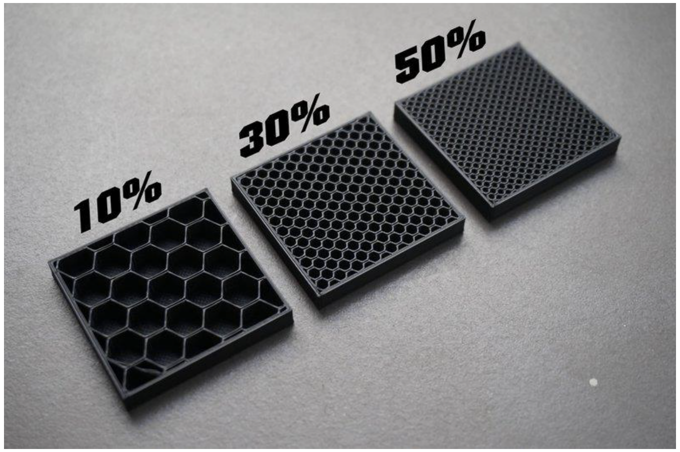
- Infill structures used to reduce weight / time-cost
- Different infill structures are selected heuristically
- Problems:
  - Limited types of patterns
  - Not optimized according to different problems



Regular Uniform Infill



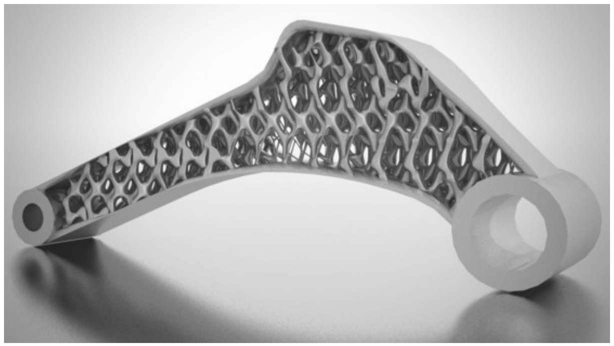
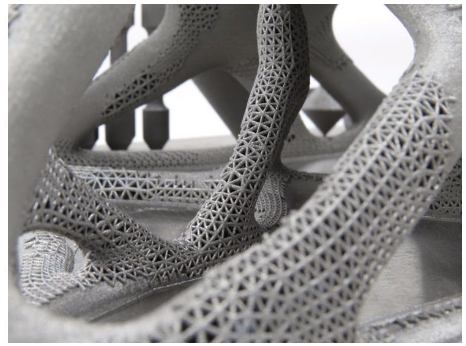
Different Patterns



Different Volumes (by Scaling)

Optimization on Interior Structures

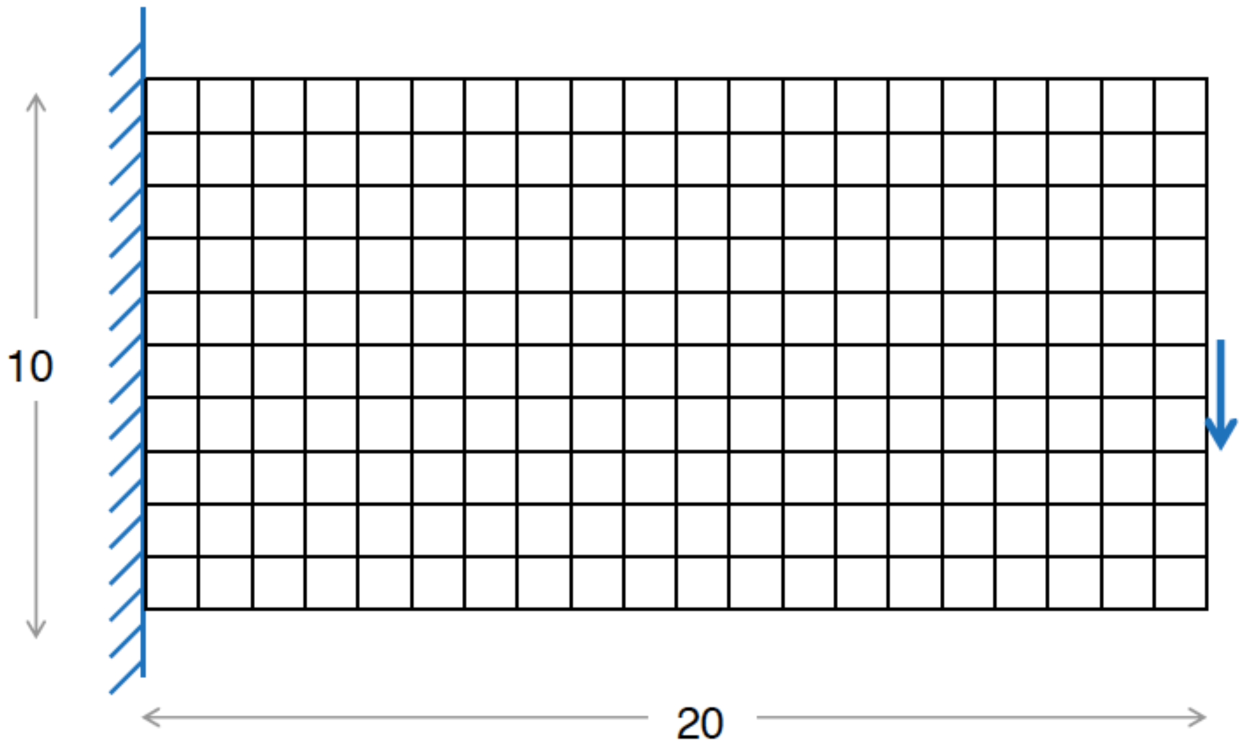
- A lot of recent work in topology optimization

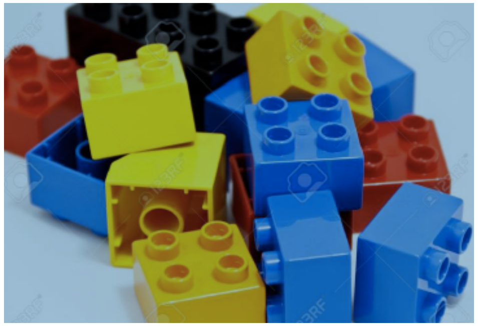




- Problem: Manufacturability? Especially for infill
- Our solution: by restricting topology optimization to generate structures in a manufacturability-ensured space
  - Use of grid refinement and grid-to-cell operators to efficiently perform the optimization
  - Optimization of mechanical stiffness and static stability effectively and efficiently

Framework of Topology Optimization

- Design a stiffest shape by placing 60 LEGO blocks
  - 60? as smallest number as possible (i.e., lightweight demand)

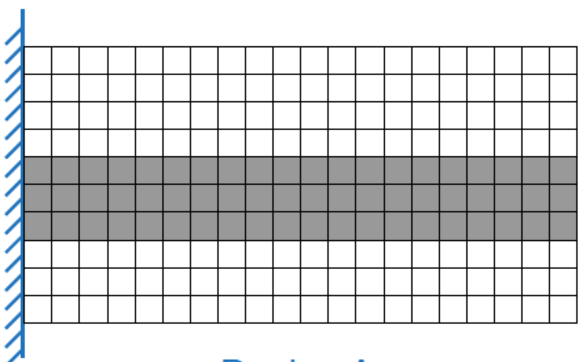




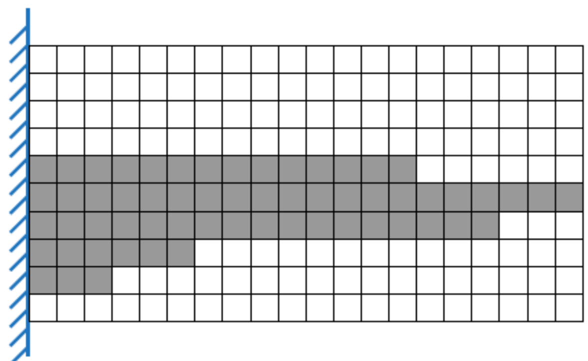
Bone Chair
by Joris Laarman

A Toy Example: Possible Choices

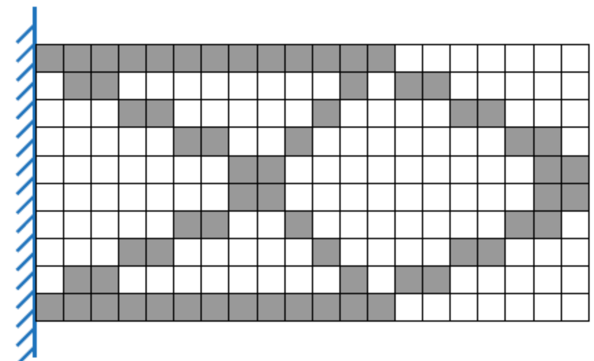
- Number of possible designs: $C_{200}^{60} = 7.04 \times 10^{51}$
- Which one has the highest stiffness?



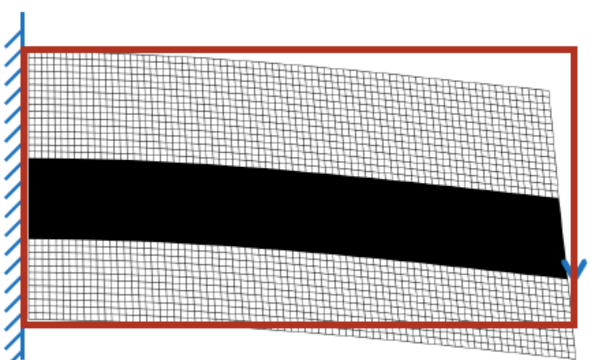
Design A



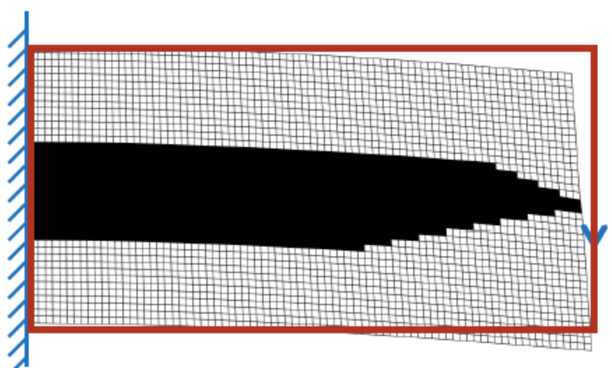
Design B



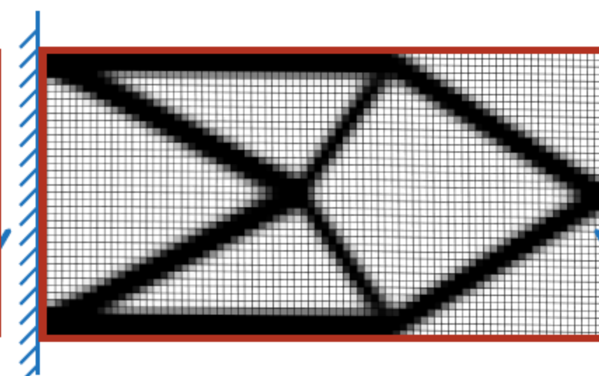
Design C



Design A



Design B

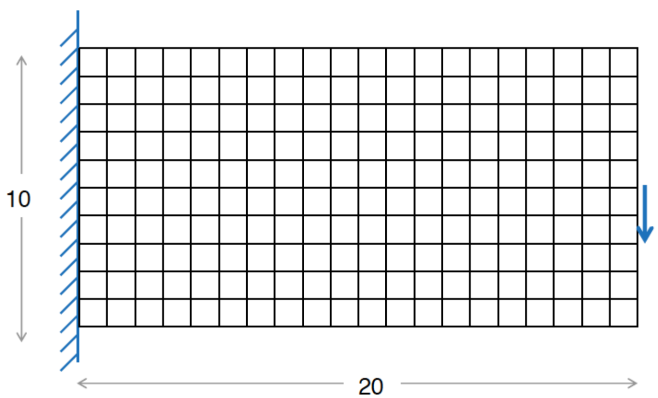


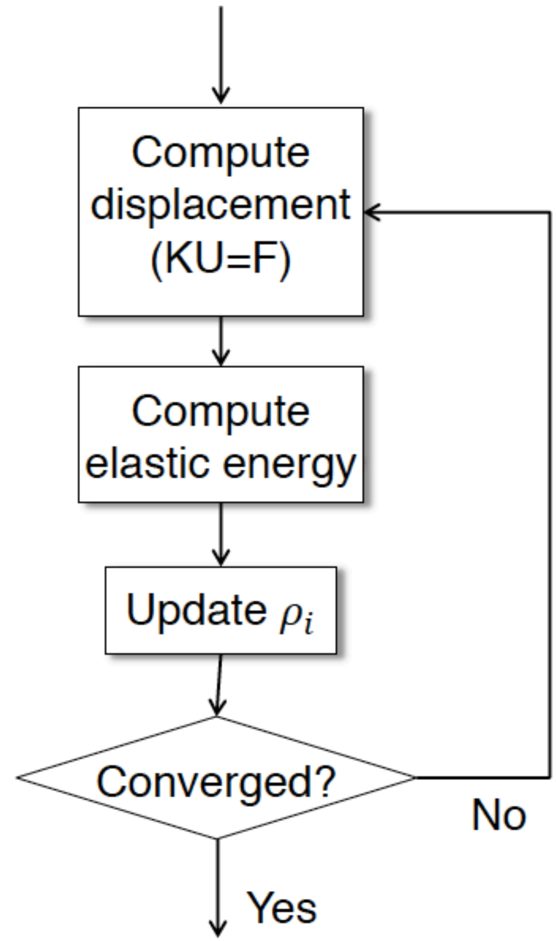
Design C

Framework of Topology Optimization

- Topology optimization for minimizing the strain energy

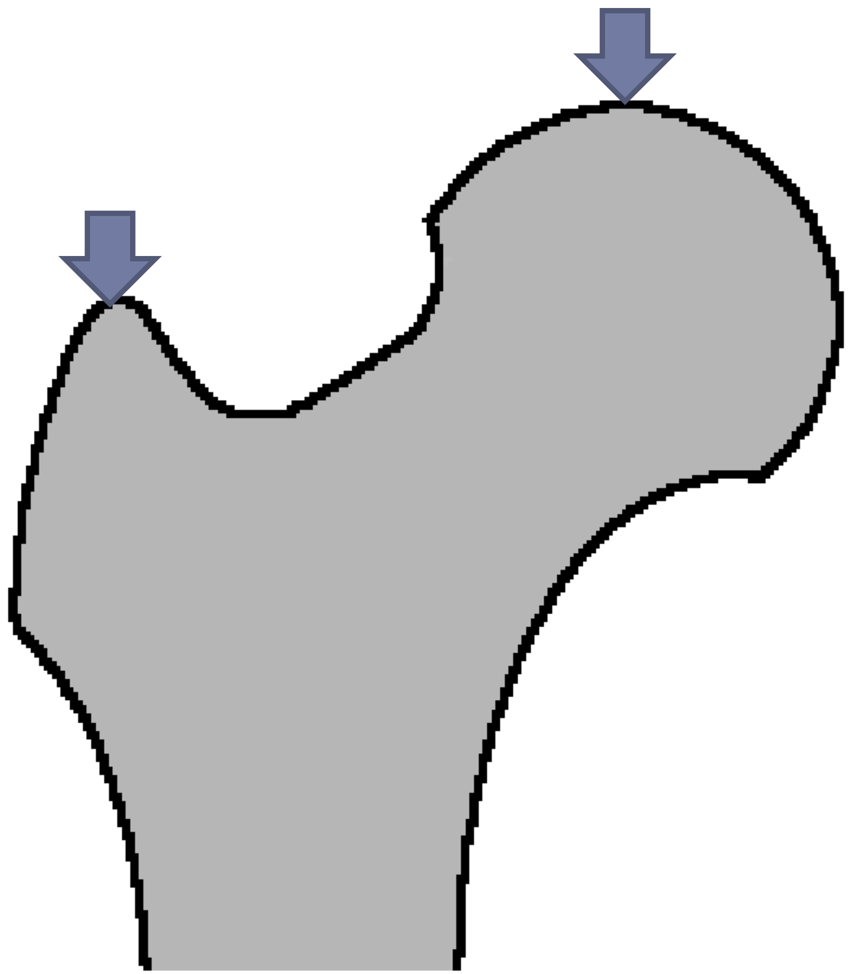
$$
\begin{aligned}
\underset{\beta}{\text{minimize}} \quad & E = \frac{1}{2} u^T K(\rho) u, \\
\text{subject to} \quad & K(\rho) u = f, \\
& V(\rho) = \sum_e \rho_e \leq V^*, \\
& \rho_e(\beta) = \begin{cases} 1.0 & e \in solid, \\ \rho_{min} & e \in cavity, \end{cases} \\
& \beta_c \in \{0, 1\}, \forall C.
\end{aligned}
$$





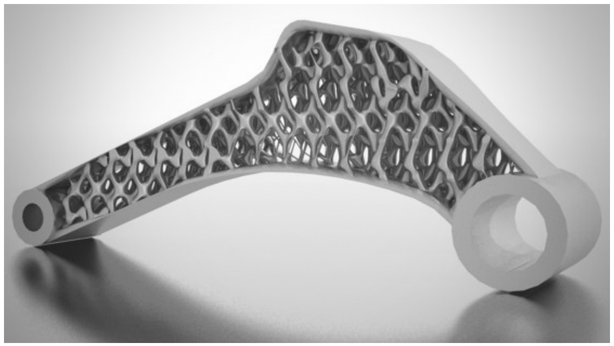
Illustration

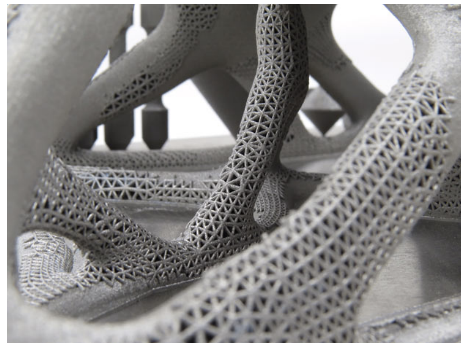
Illustration of Topology Optimization



Optimization on Rhombic Structures

- A lot of recent work in topology optimization





- Problem: Manufacturability? Especially for infill
- Our solution: by restricting topology optimization to generate structures in a manufacturability-ensured space
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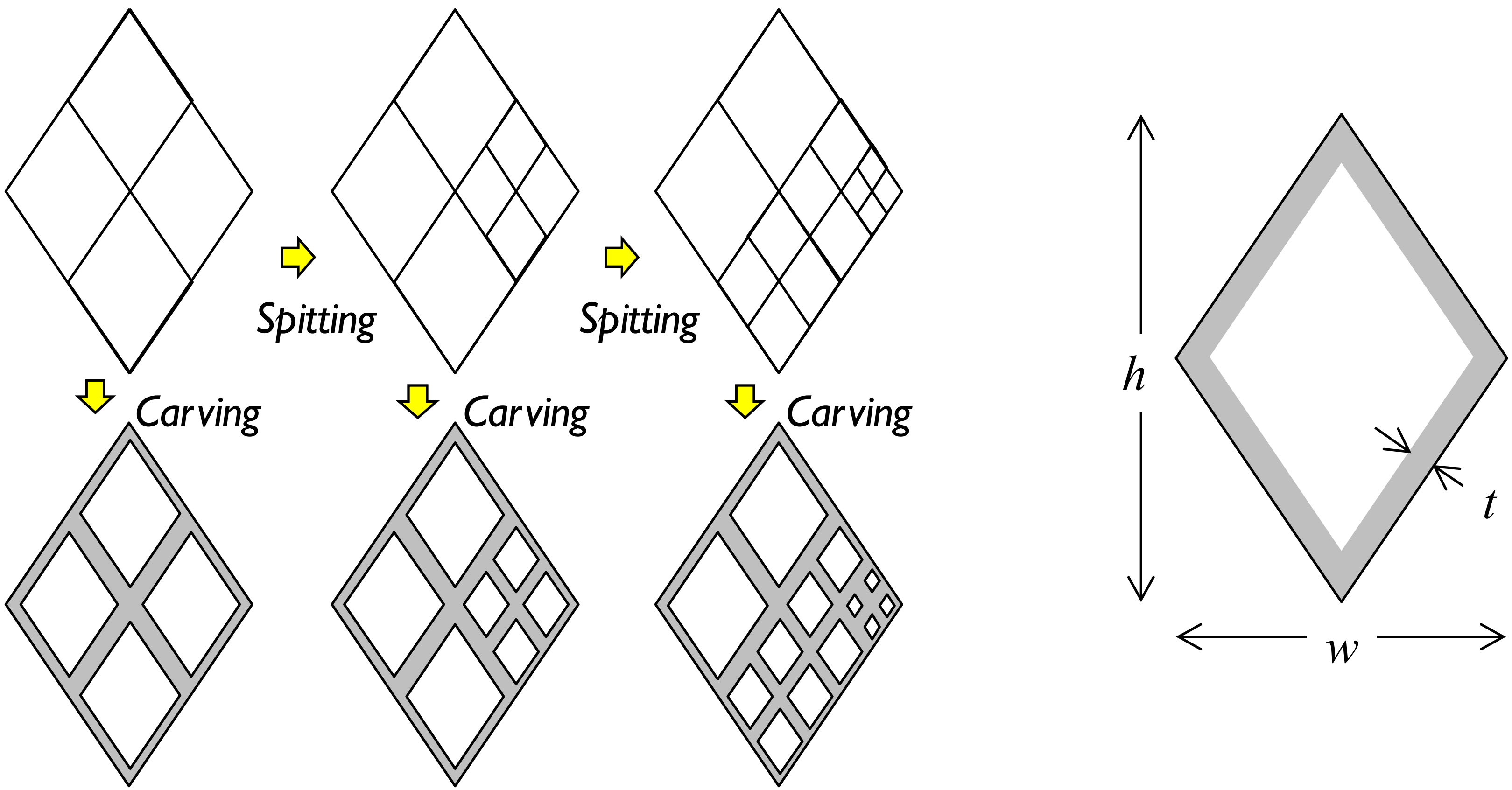
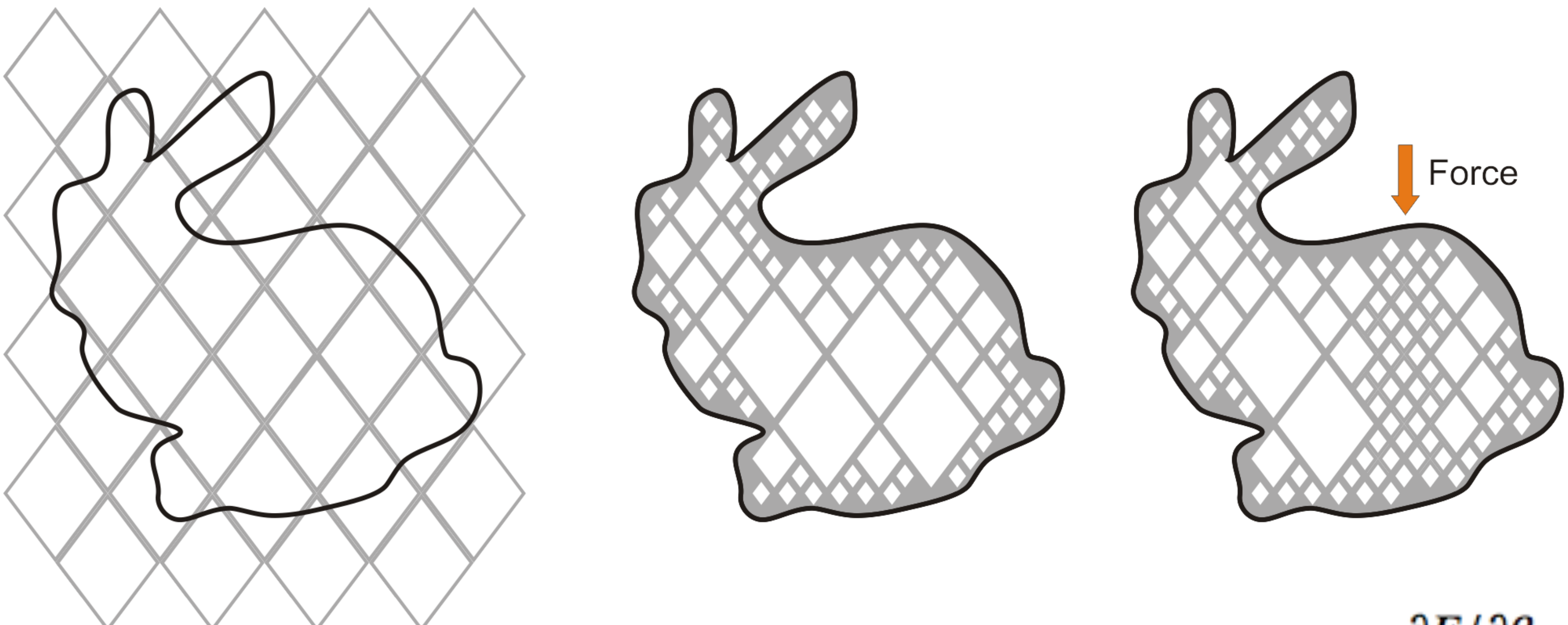
Adaptive Subdivision and Dual-Rep.

- Manufacturability can be ensured by the geometric setting: *Slope Angle* and *Wall Thickness*



Flow of Optimization by Refinement



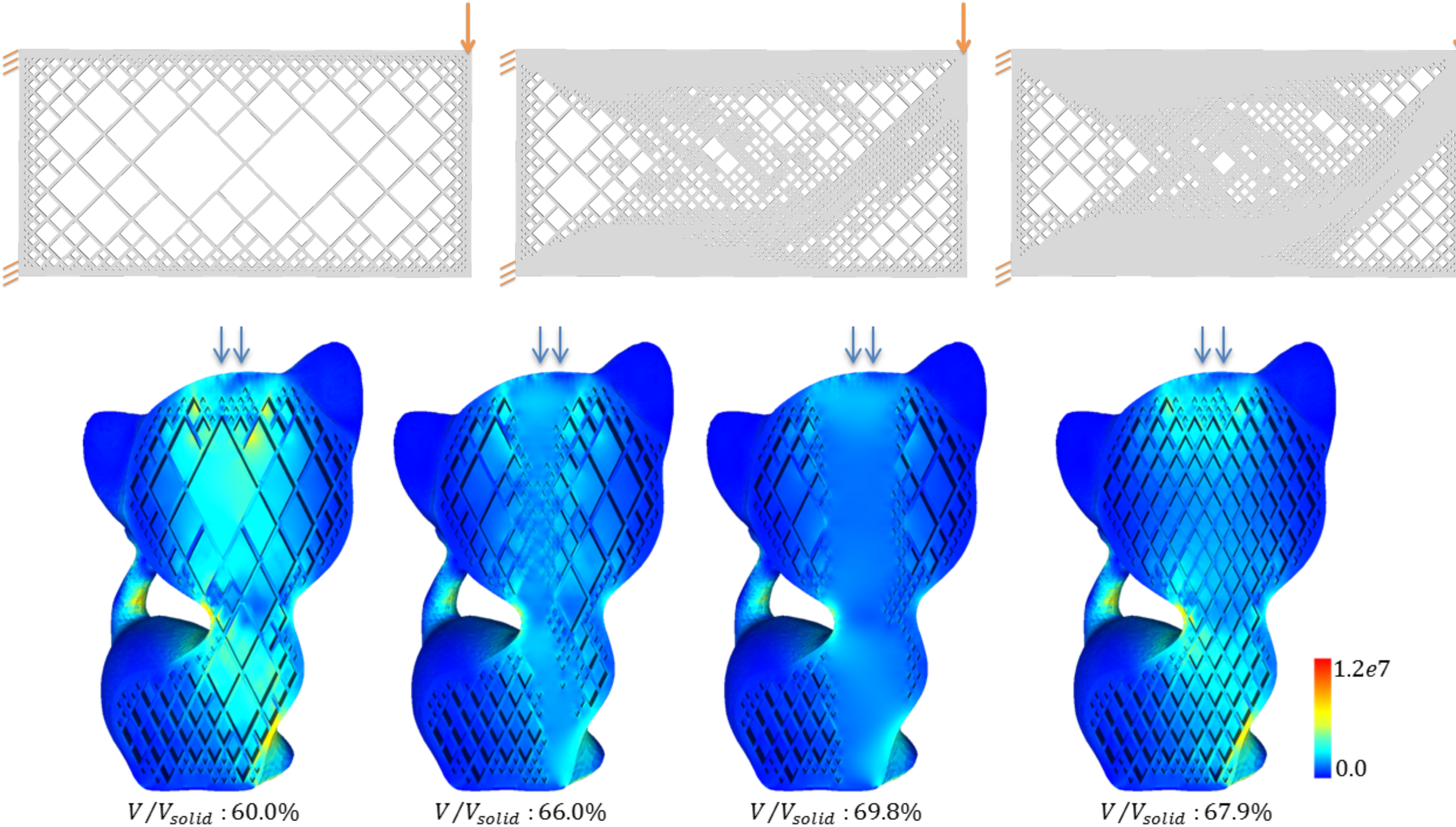
- An iterative process by sensitivity analysis:

$$G_c = \frac{-\partial E / \partial \beta_c}{\partial V / \partial \beta_c}$$

1. Finite Element Analysis of elasticity
2. Evaluate the sensitivity
3. Update the rhombic structure by subdividing selected cells

Top 2% with high sensitivity

Self-Supporting Infill Optimization

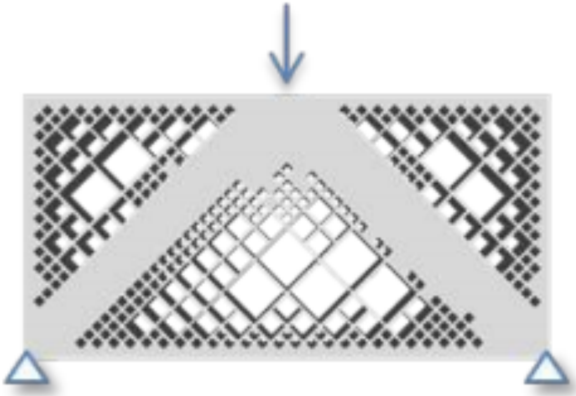


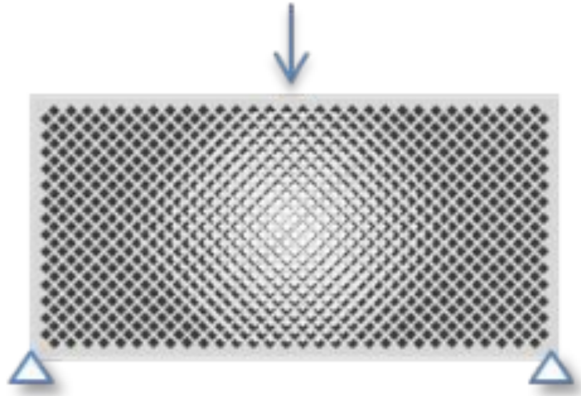
Uniform Cells

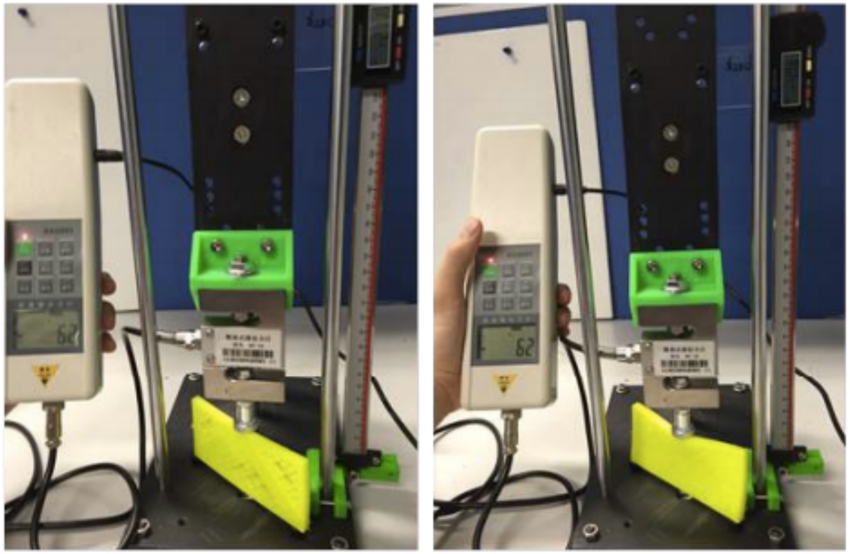
Optimization of Infill Structures

- Physical Tests

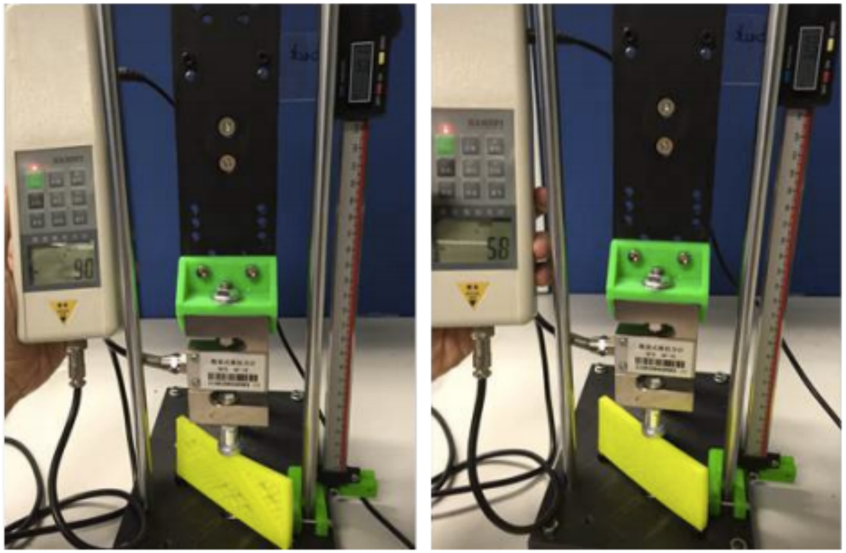
(for comparison)







Applying the same loading (2.11 vs 4.08mm)

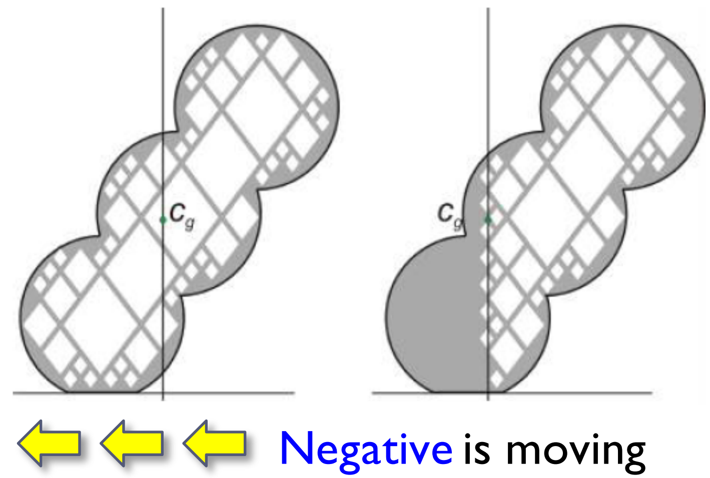


Under the same displacement (3mm)

Formulation: Static Stability

- Defining an energy to move the center of gravity, $\mathbf{c}_g$, into the convex hull of contacted point on the ground
- A weak form formulation using center of convex hull, $\boldsymbol{c}_h$

$$\underset{\beta}{\text{minimize}} \quad E_s = \|(c_g - c_h)^{\perp g}\|_2^2,$$
$$\text{subject to} \quad \beta_c \in \{0, 1\}, \forall C.$$



The gradient

$$\frac{\partial E_s}{\partial \beta_c} = \frac{2m_c}{m_c + m_g} (c_g - c_h)^{\perp g} \cdot (c_c - c_g)^{\perp g}$$

Negative is moving towards the center $\boldsymbol{c}_h$

Sort candidate cells in ascending order

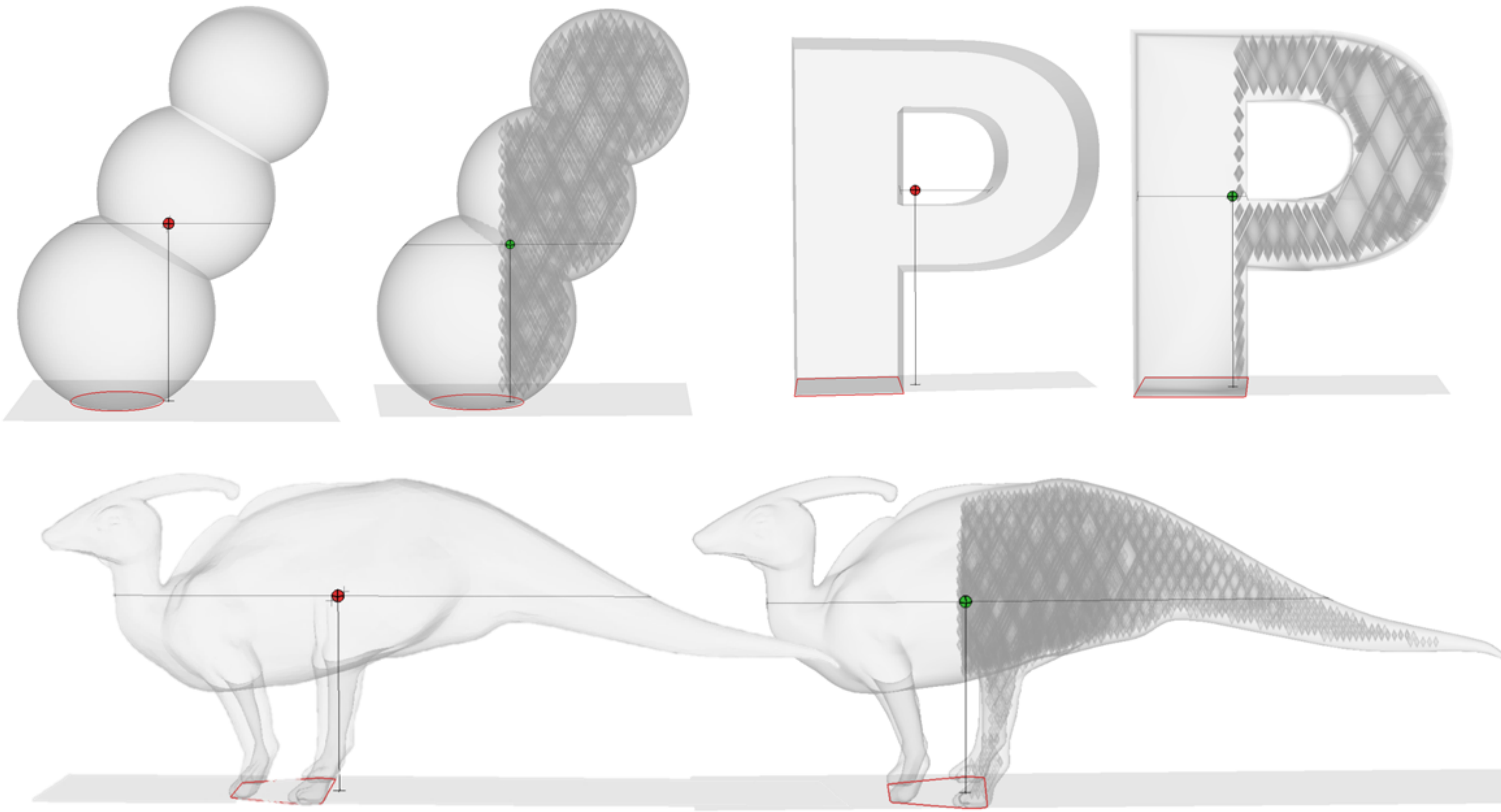
Refine one by one

$\boldsymbol{m}_c$: the center of mass after refining a considered cell

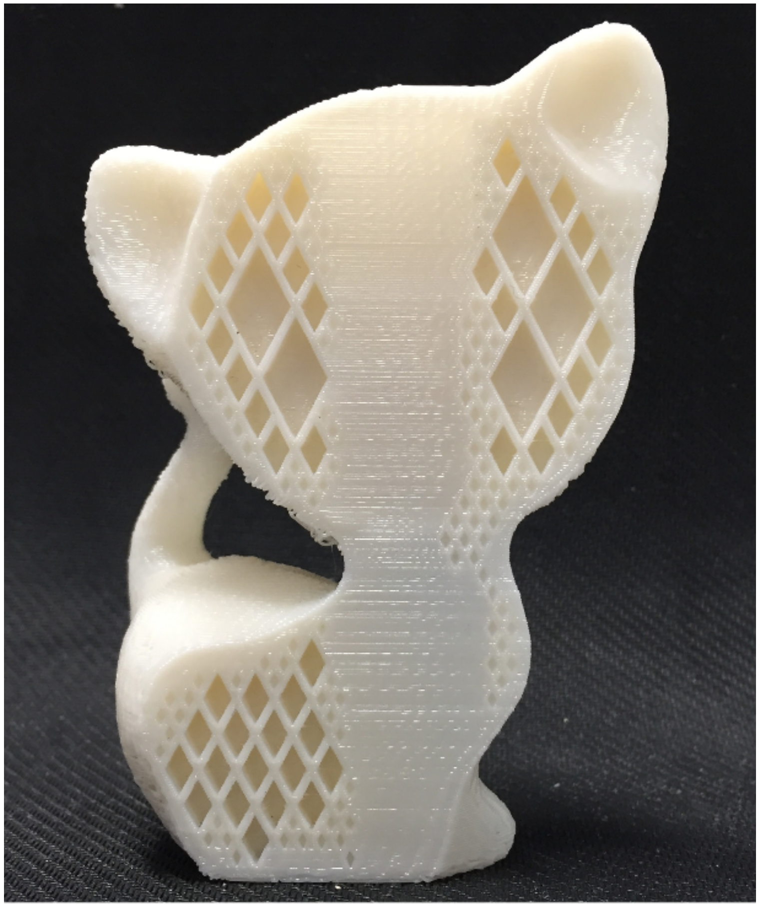
$\boldsymbol{m}_g$: the current center of mass

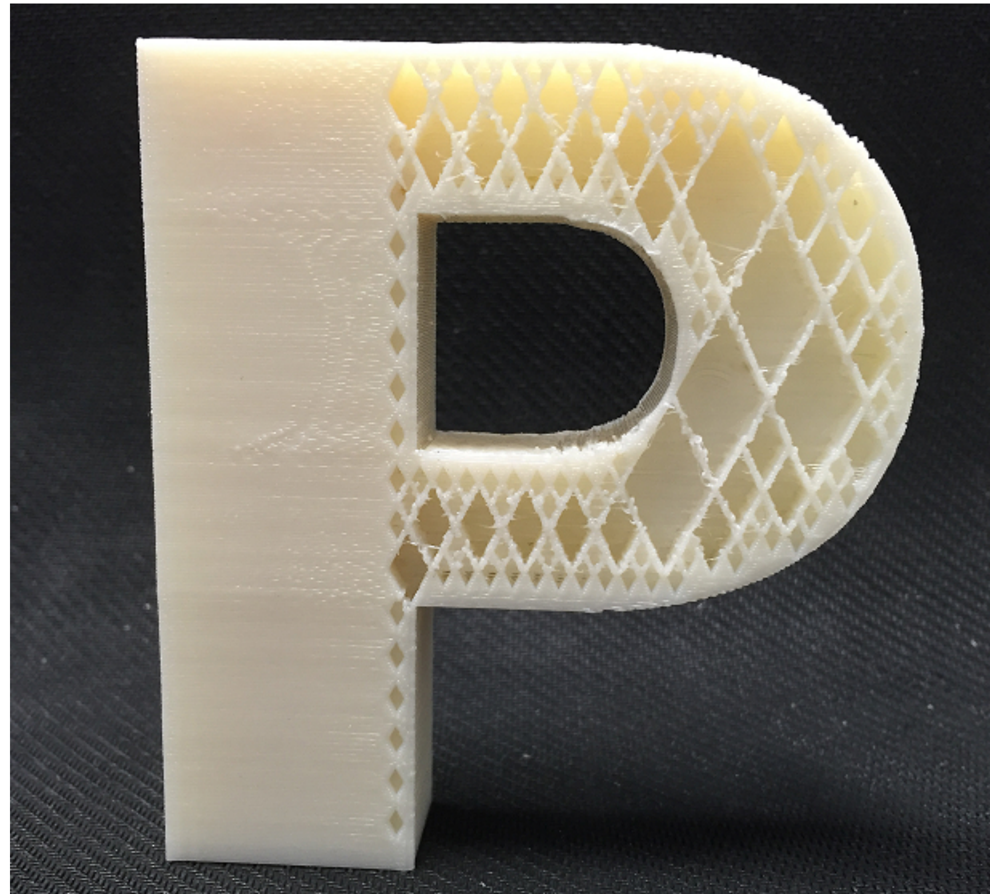
$\boldsymbol{c}_c$: the geometric center of the cell under consideration

Infill Optimization for Static Stability



Self-Supporting Infill Optimization





Jun Wu, Charlie C.L. Wang, Xiaoting Zhang, and Rudiger Westermann, "Self-supporting infill optimization on rhombic cells", *Computer-Aided Design*, 2016.

Conclusion Remarks

- An infill optimization approach ensures manufacturability explicitly
  - By restricting optimization to generate rhombic structures
  - Overhang constraint and minimal-wall constraint are seamlessly satisfied
  - Validated in both mechanical stiffness and static stability problems
- Open questions:
  - Greedy approach – no guarantee of global optimality
  - What about allowing boundary to deform? within controlled region?
  - More sparse self-supporting structures?
  - Current - minimizing compliance, how about other objectives?
    - Maximizing compliance?
    - Constraints on maximum stress?