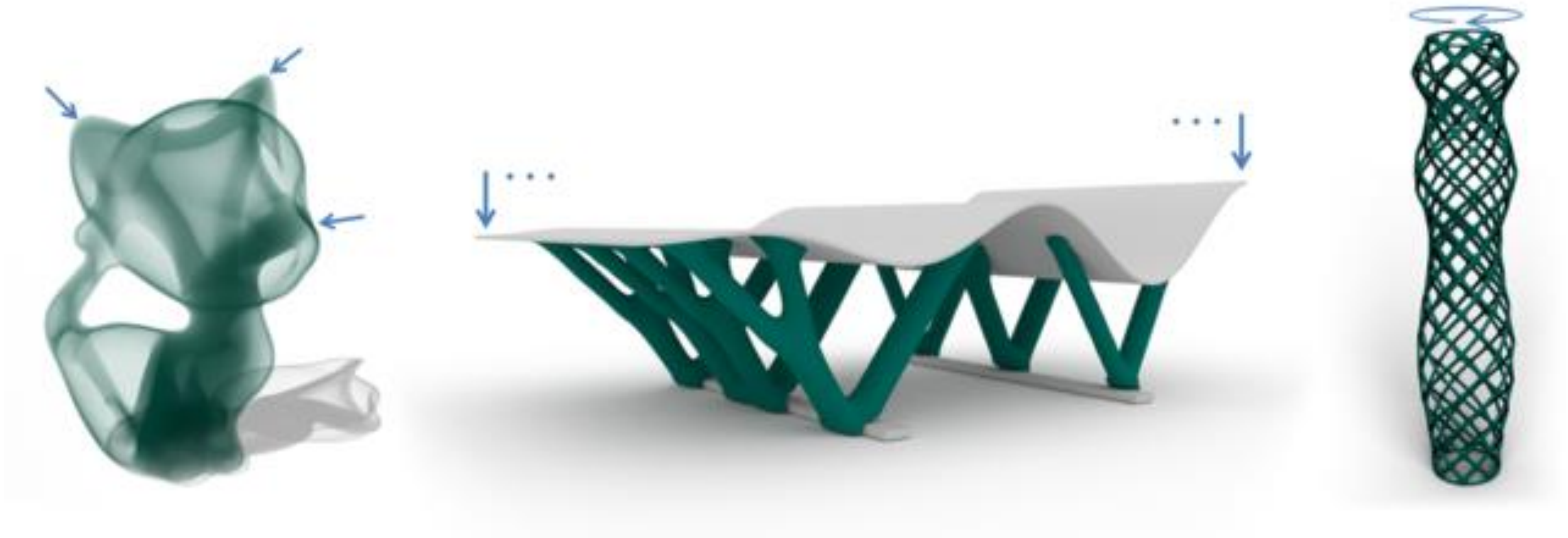
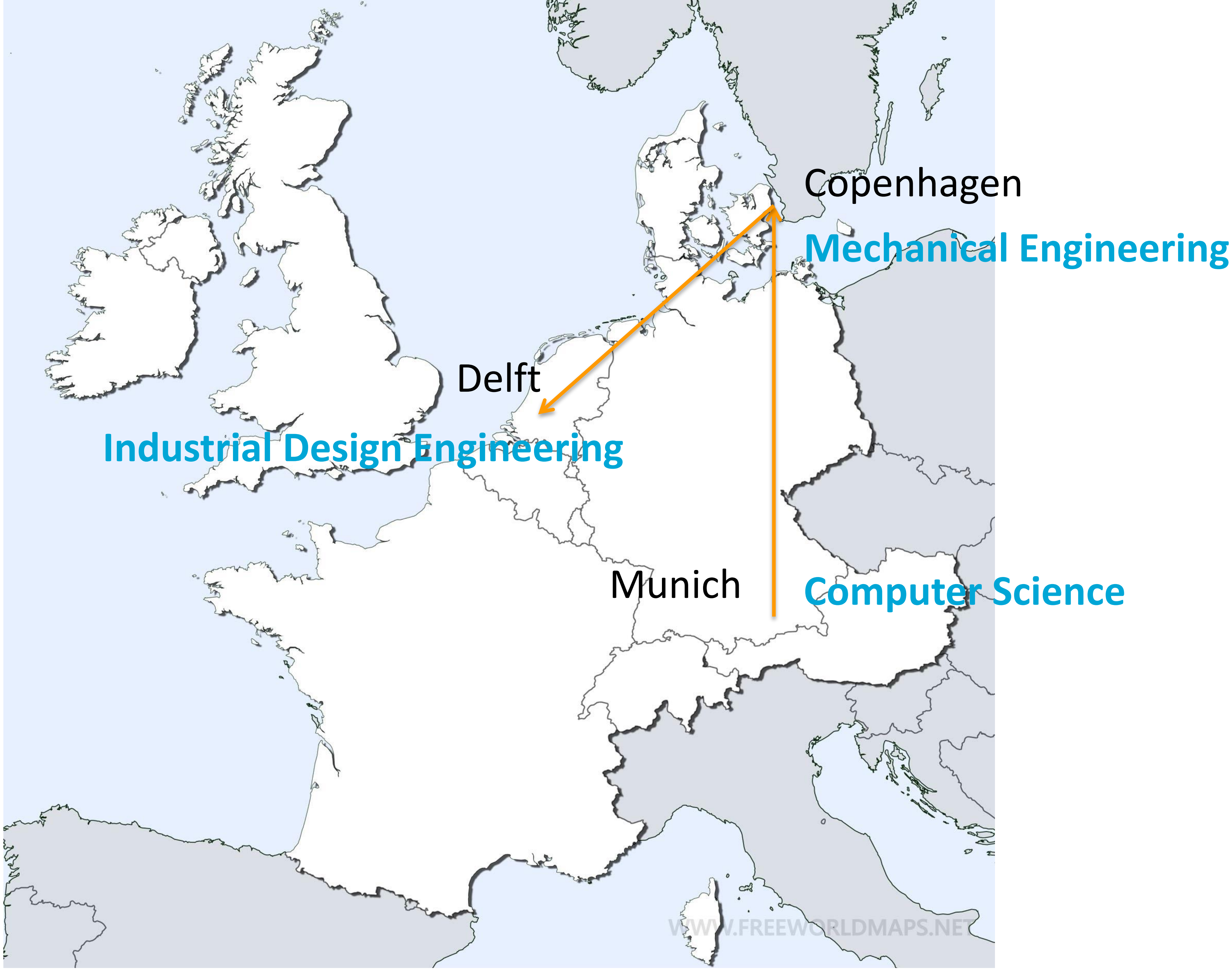
Topology Optimization for Computational Fabrication



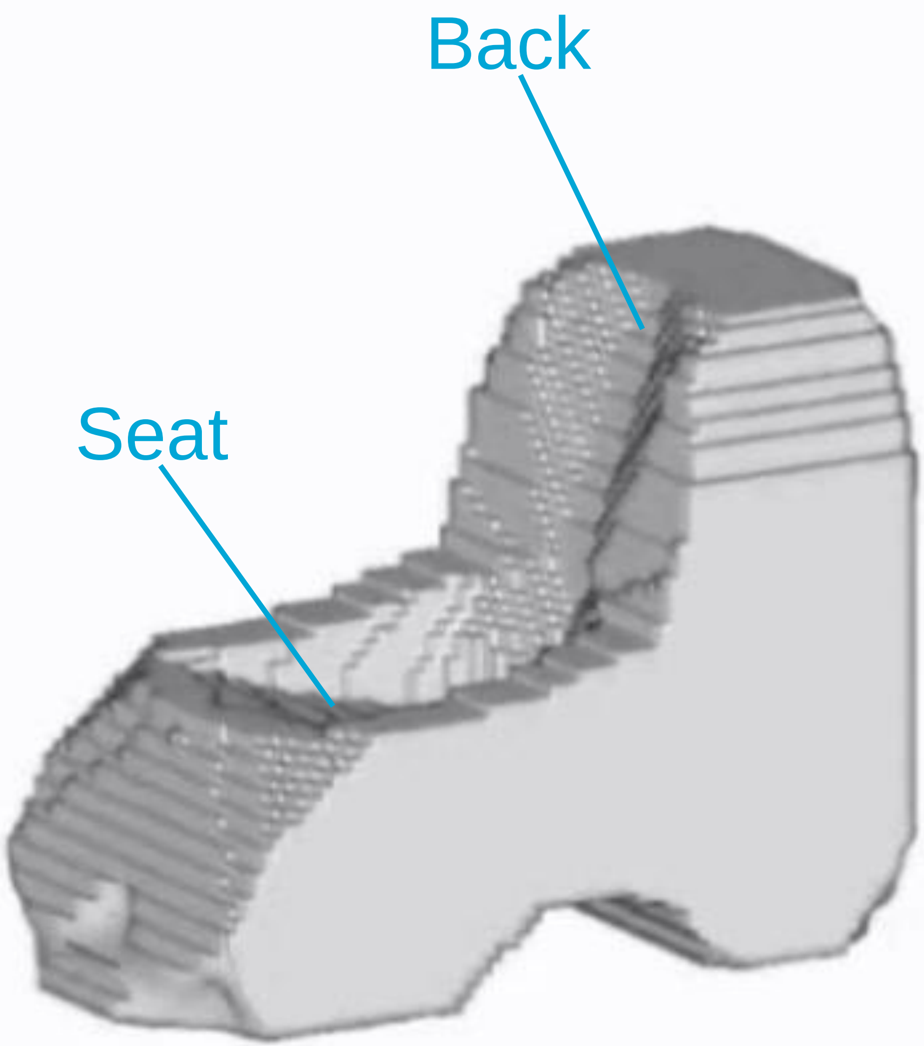
Jun Wu

Depart. of Design Engineering

j.wu-1@tudelft.nl


Copenhagen
Mechanical Engineering
Delft
Industrial Design Engineering
Munich
Computer Science
WWW.FREEWORLDMAPS.NET

Bone Chair by Joris Laarman



Optimization of Bone Chair
by Lothar Harzheim & Opel GmbH


www.jorislaarman.com

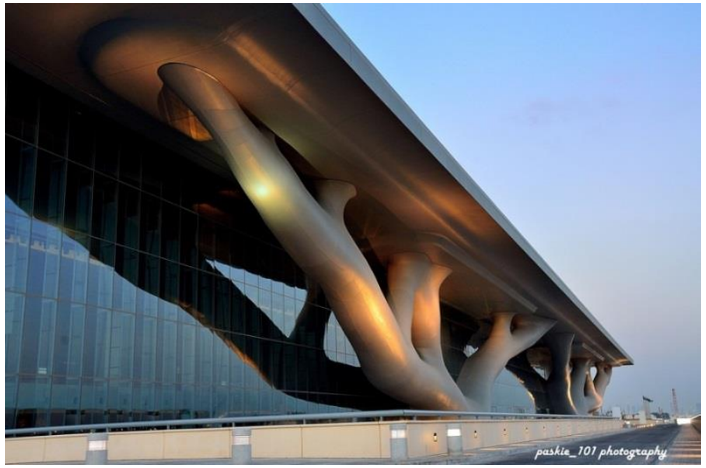
Schedule

- Basics of Topology Optimization (45’)
- Break (15’)
- Topology Optimization for Additive Manufacturing (45’)
- Break (15’)
- Exercises and Assignment (45’)

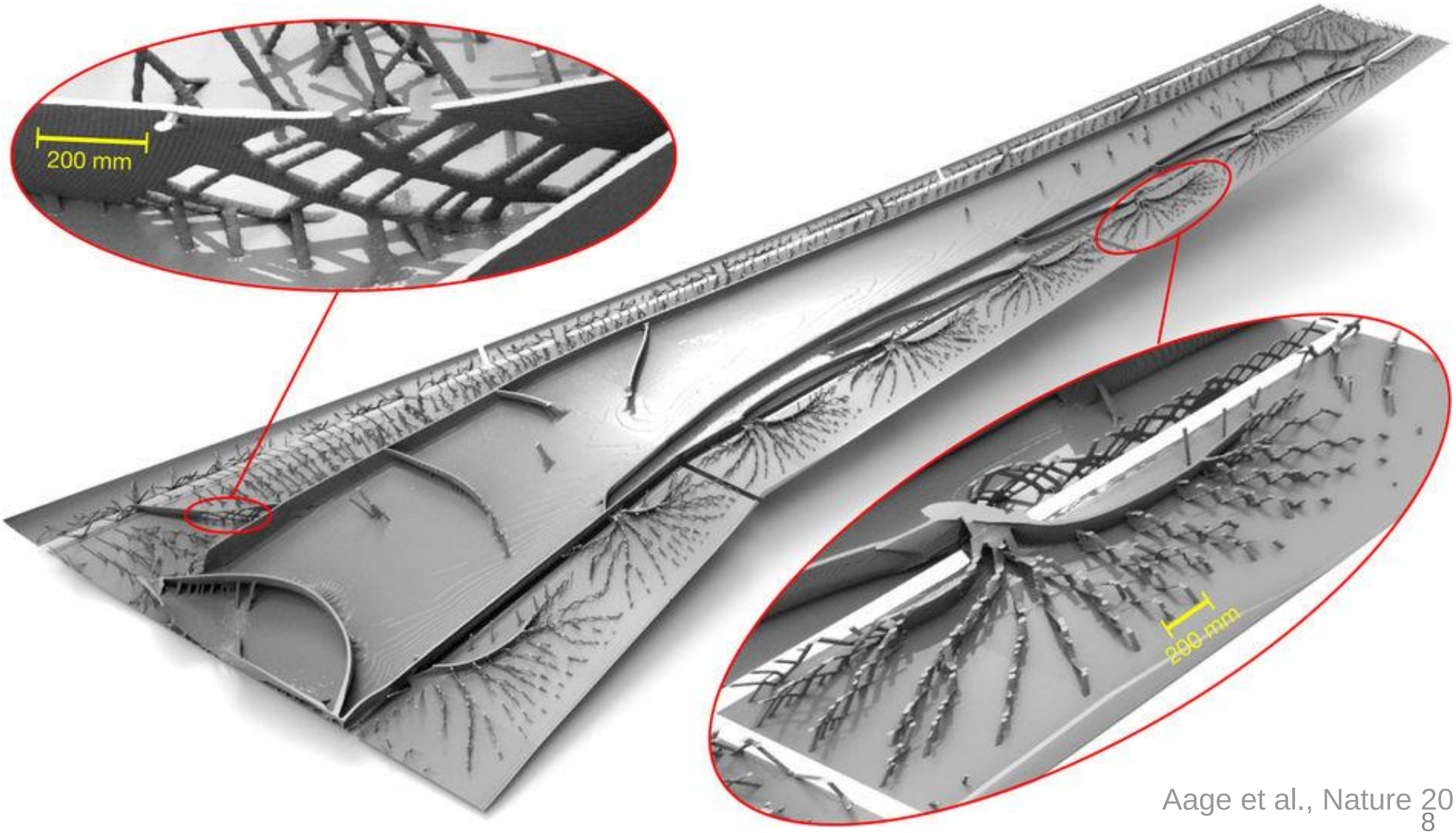
Topology Optimization Examples

Frustum Inc.

Airbus APWorks, 2016

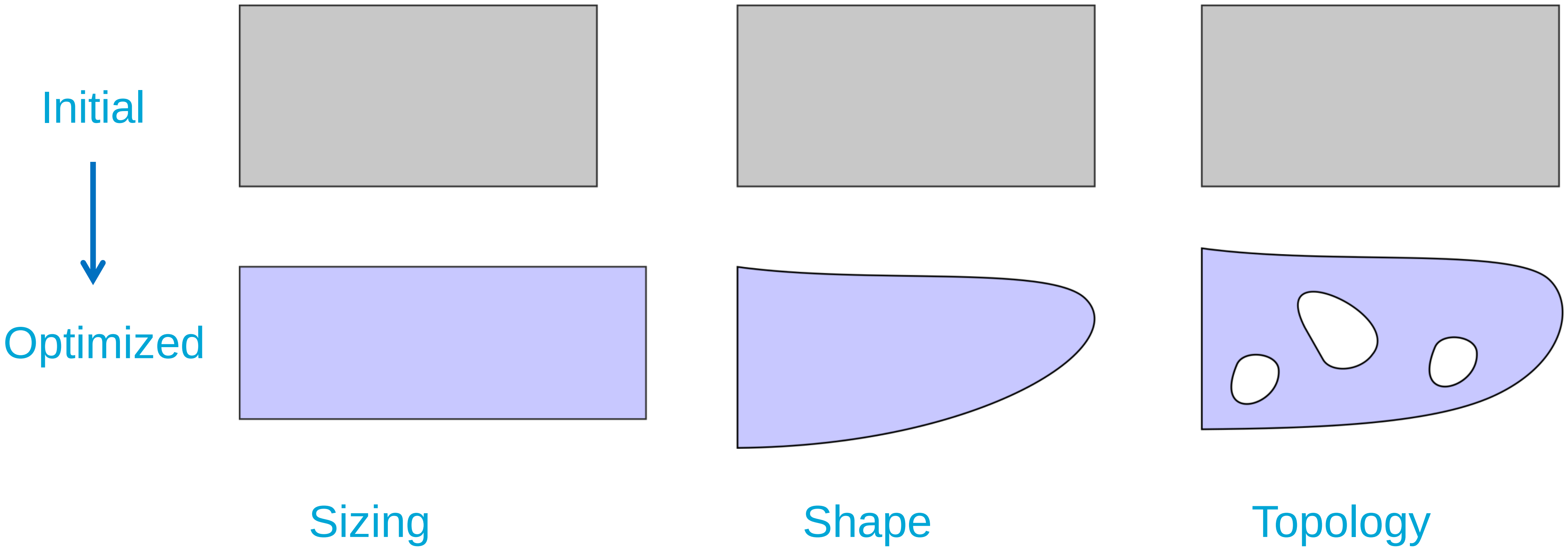


Qatar national convention



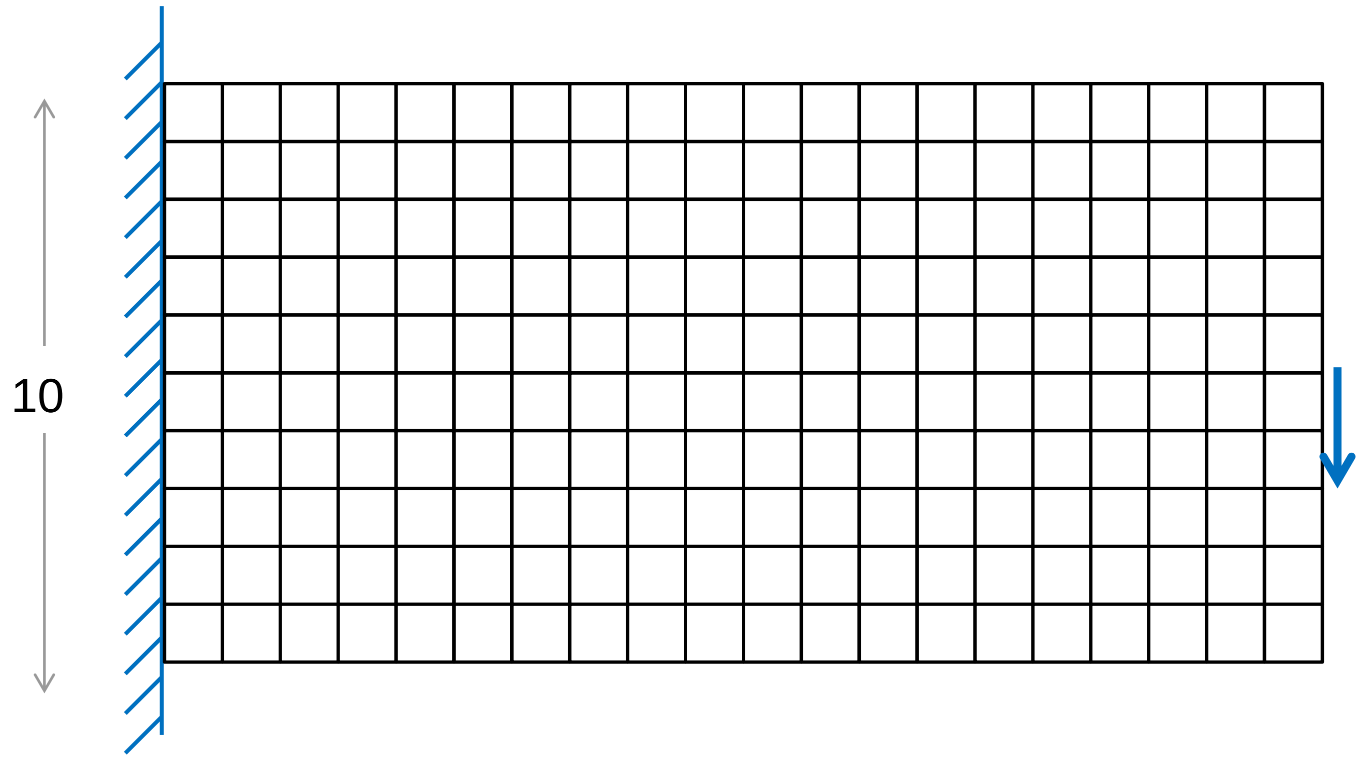
Aage et al., Nature 2017

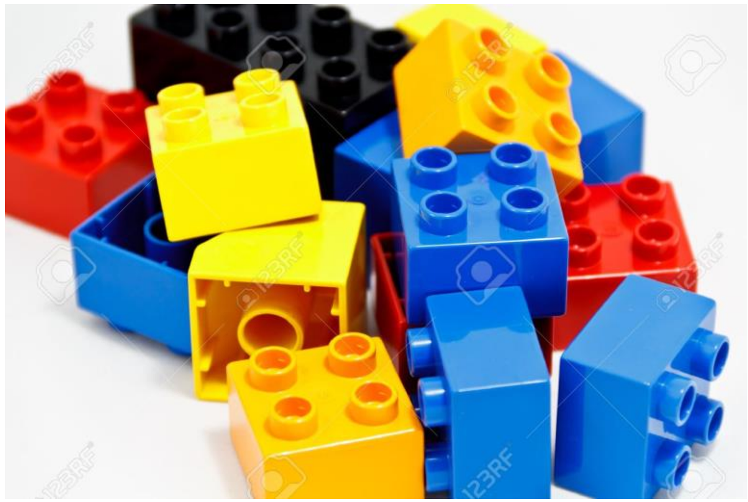
Classes of Structural optimization: Sizing, Shape, Topology



A Toy Problem

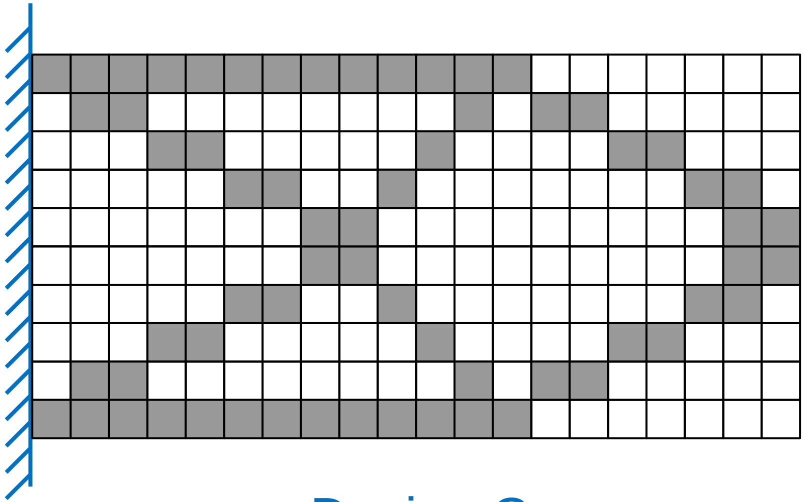
- Design the **stiffest** shape, by placing $\mathbf{60}$ Lego blocks into a grid of $\mathbf{20} \times \mathbf{10}$



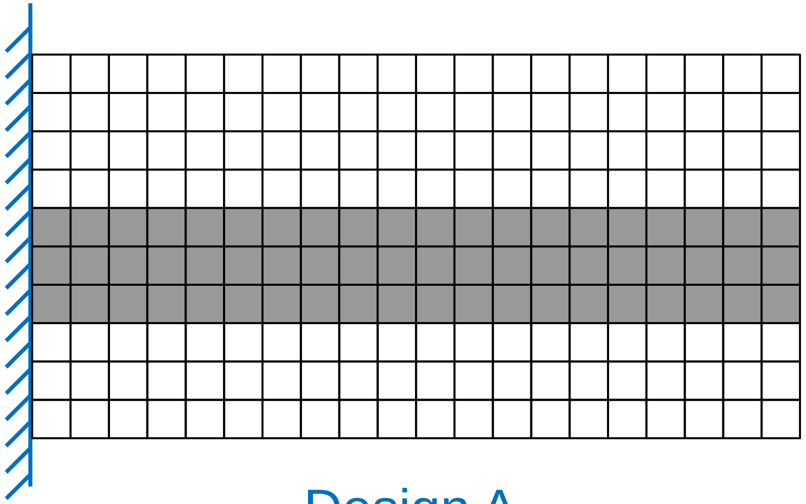


A Toy Problem: Possible Solutions

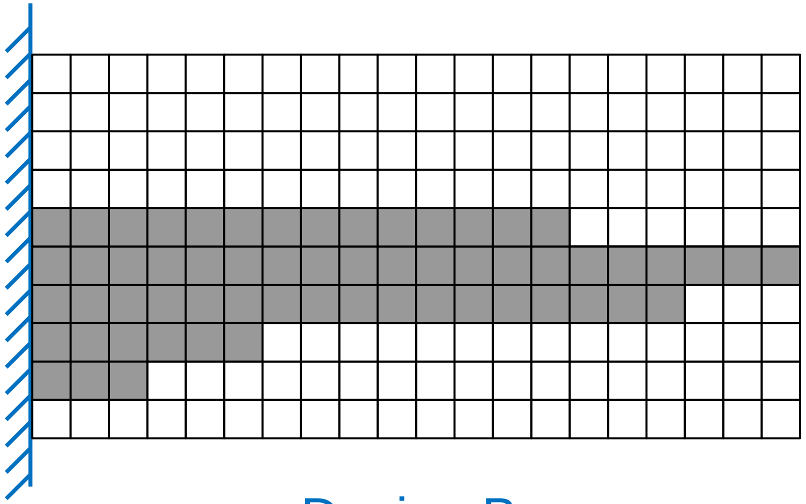
- Number of possible designs
  - $C(200,60) = \frac{200!}{60!(200-60)!} = 7.04 \times 10^{51}$
- Which one is the stiffest?



Design C



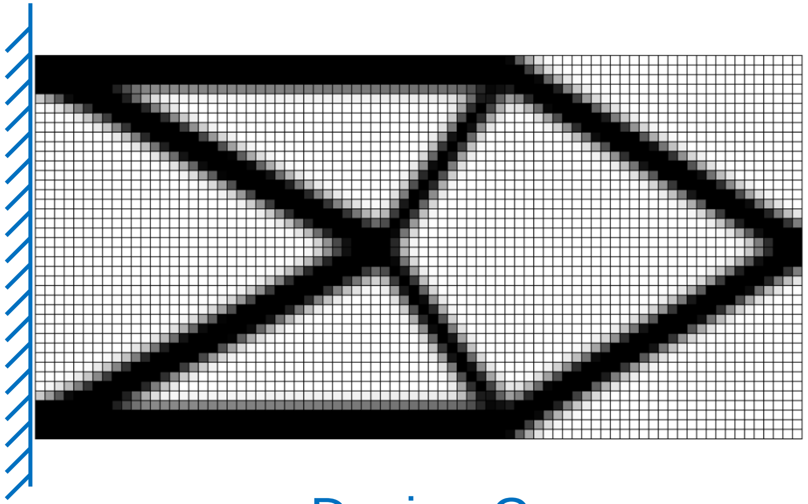
Design A



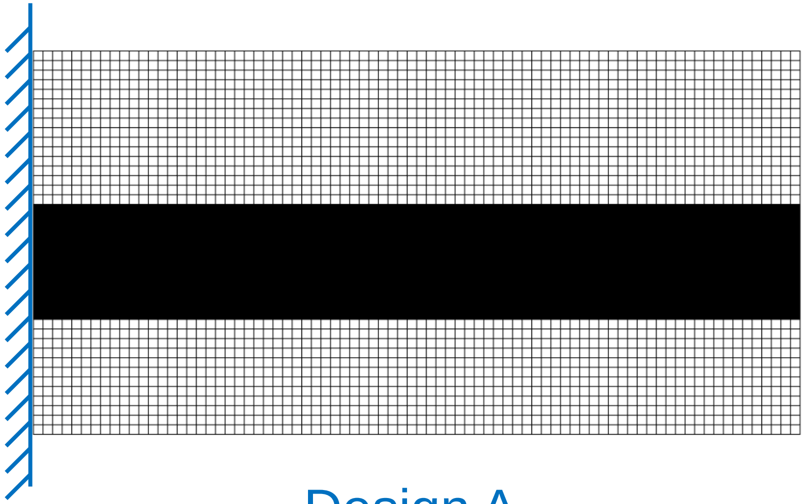
Design B

A Toy Problem: Possible Solutions

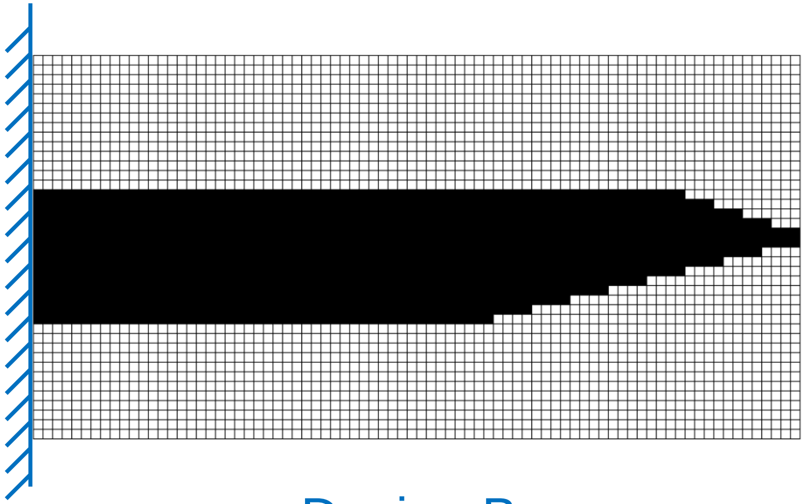
- Which one is the stiffest?



Design C



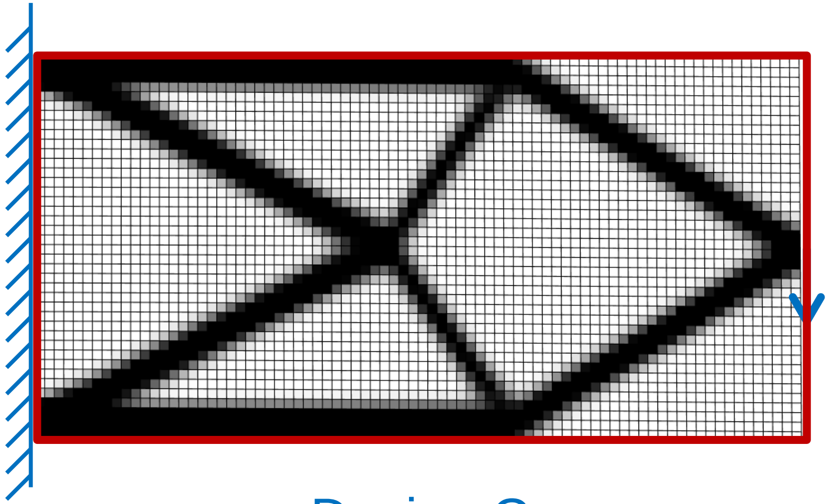
Design A



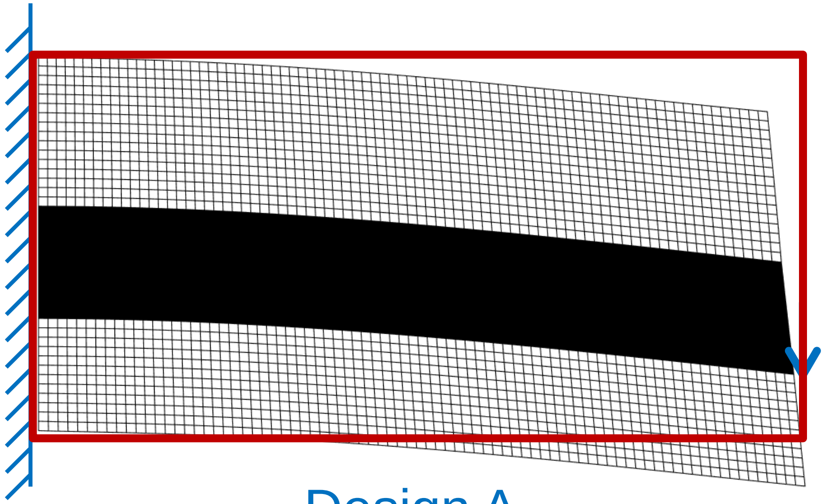
Design B

A Toy Problem: Possible Solutions

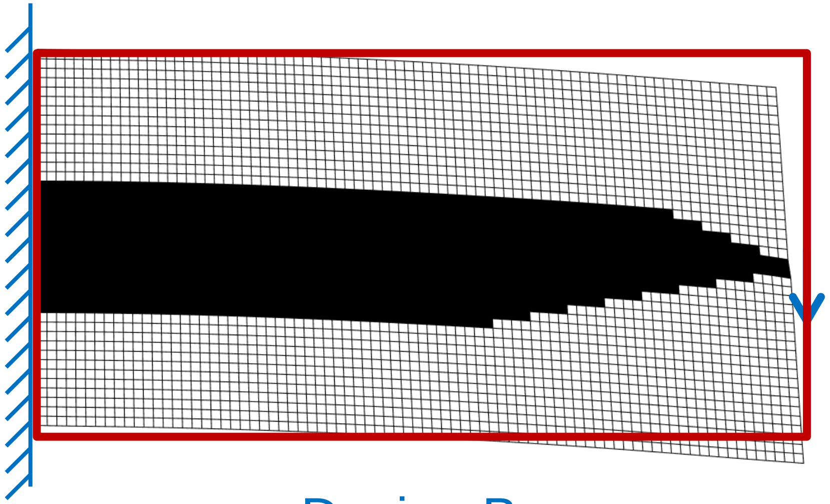
- Which one is the stiffest?



Design C

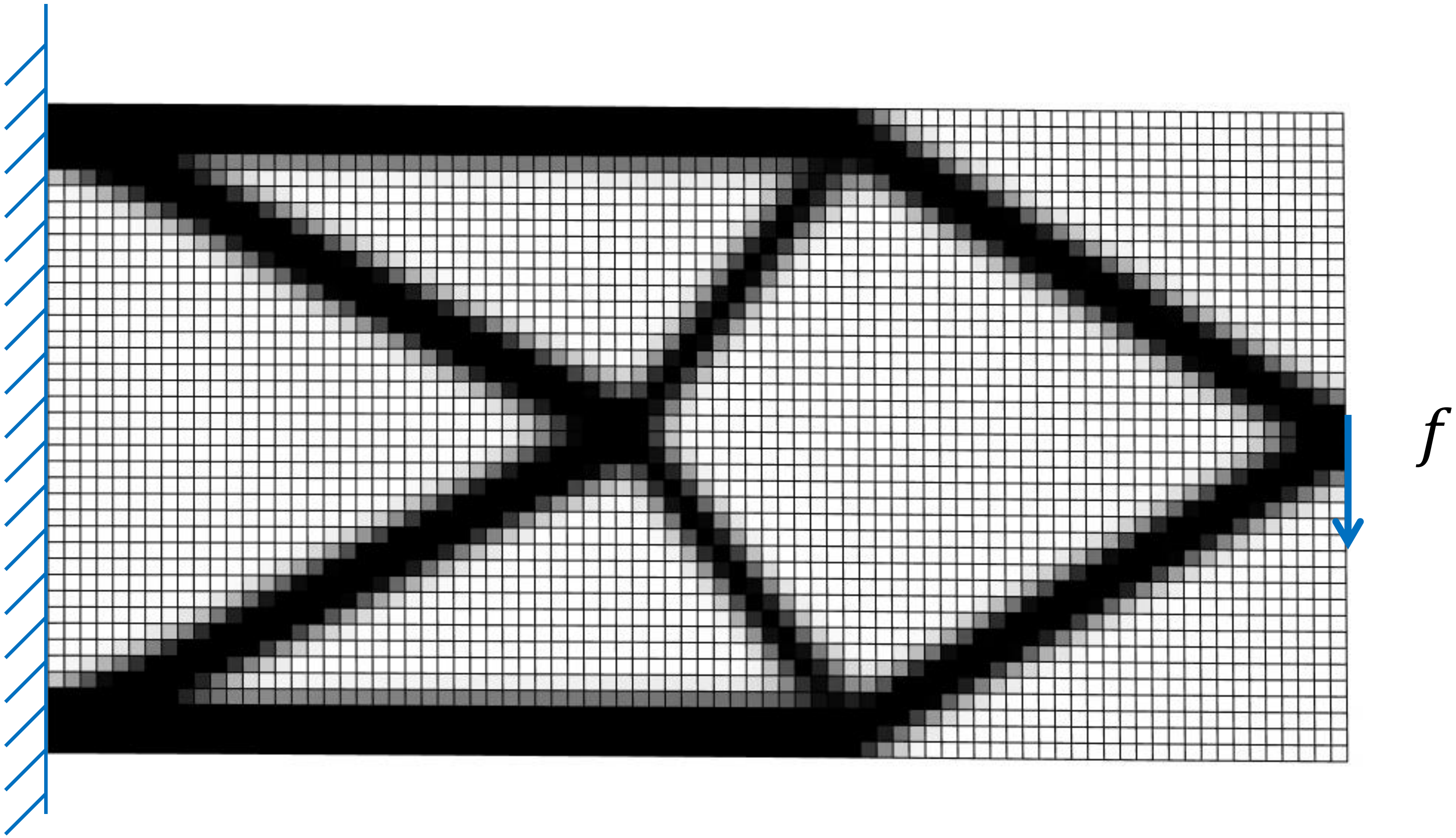


Design A



Design B

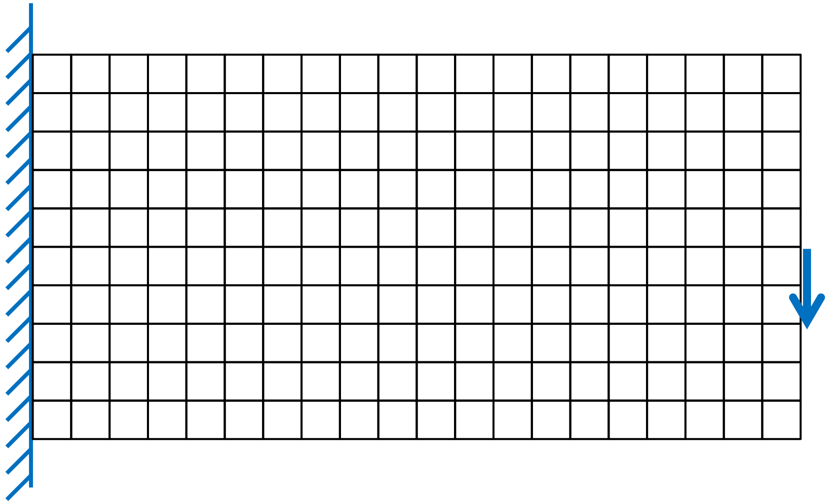
Topology Optimization Animation

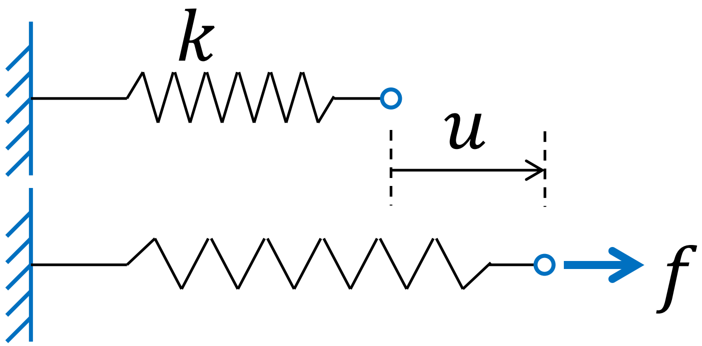


Topology Optimization

Minimize: $c = \frac{1}{2} U^T K U$ ← Elastic energy $c = \frac{1}{2} f u = \frac{1}{2} k u^2$

Subject to: $KU = F$ ← Static equation $ku = f$





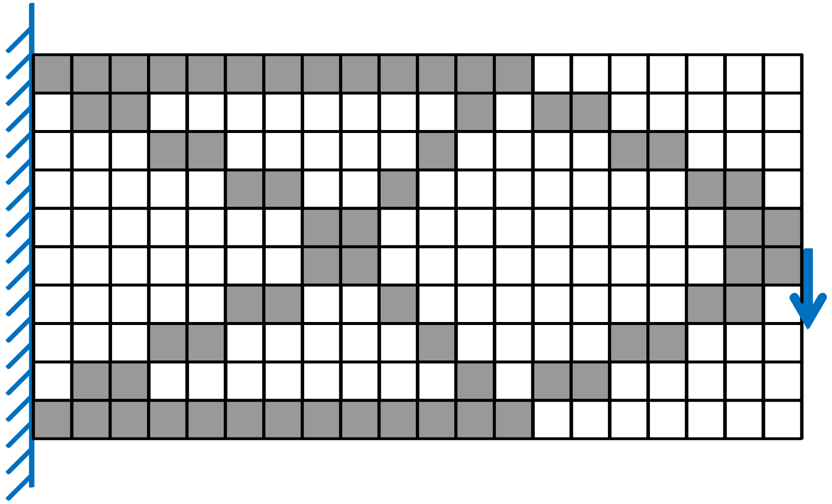
Topology Optimization

Minimize: $c = \frac{1}{2} U^T K U$ Elastic energy

Subject to: $KU = F$ Static equation

$$\rho_i = \begin{cases} 1 & \text{(solid)} \\ 0 & \text{(void)} \end{cases}, \forall i$$ Design variables

$$\mathrm{g} = \sum_i \rho_i - V_0 \leq 0$$ Volume constraint



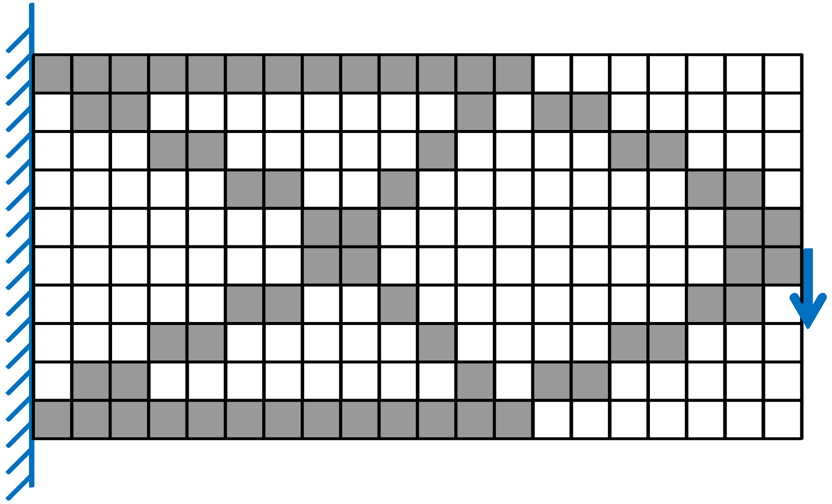
Topology Optimization

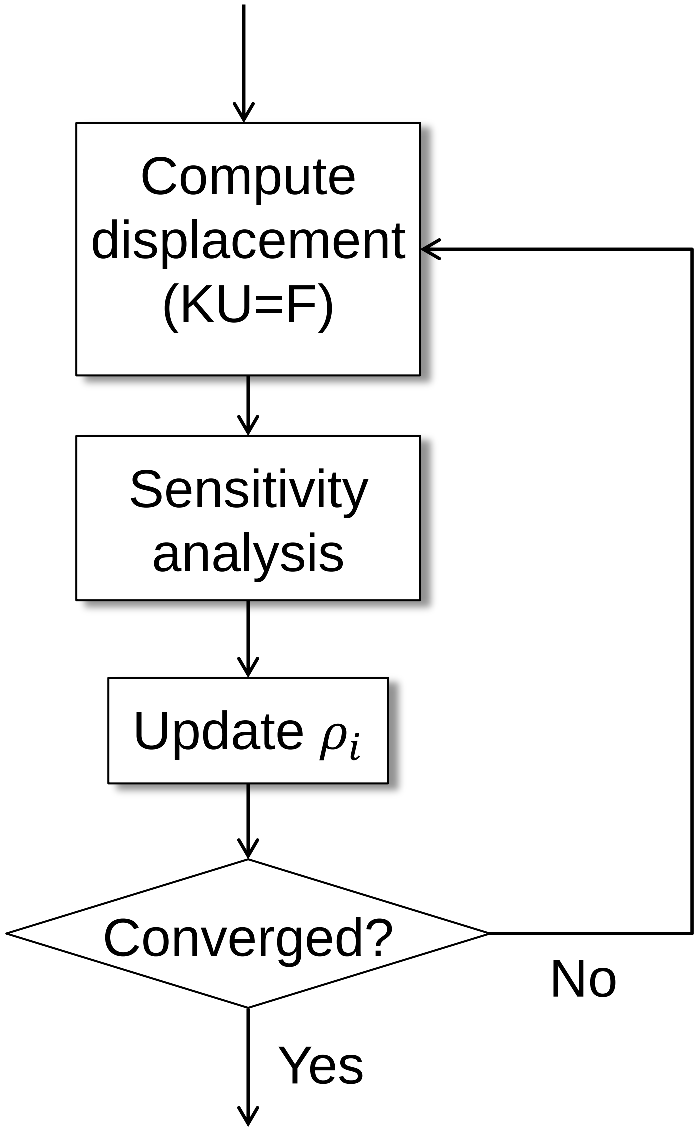
Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

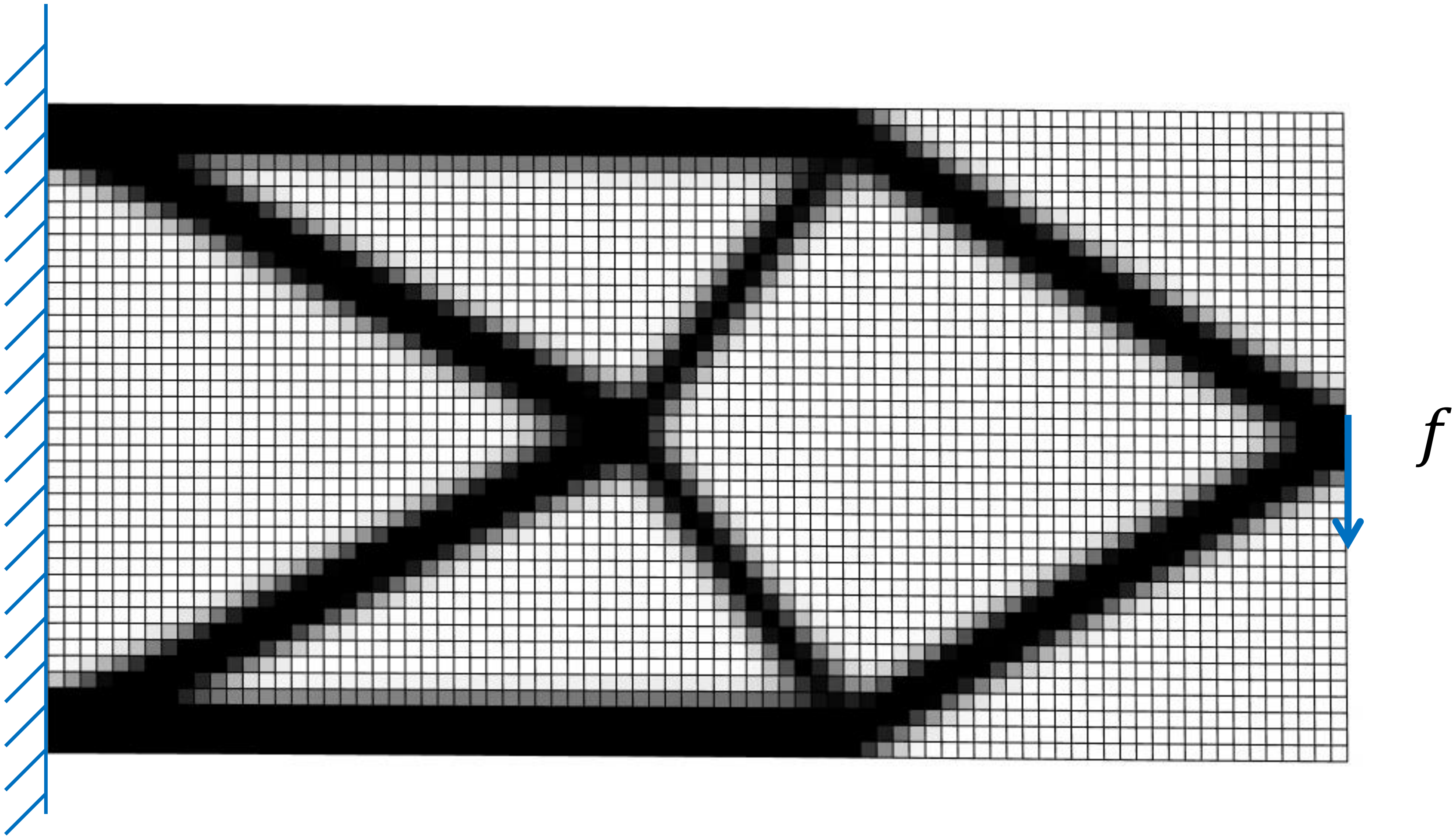
$$\rho_i = \begin{cases} 1 & \text{(solid)} \\ 0 & \text{(void)} \end{cases}, \forall i$$

$$g = \sum_i \rho_i - V_0 \leq 0$$





Topology Optimization Animation



Relaxation: Discrete to Continuous

Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

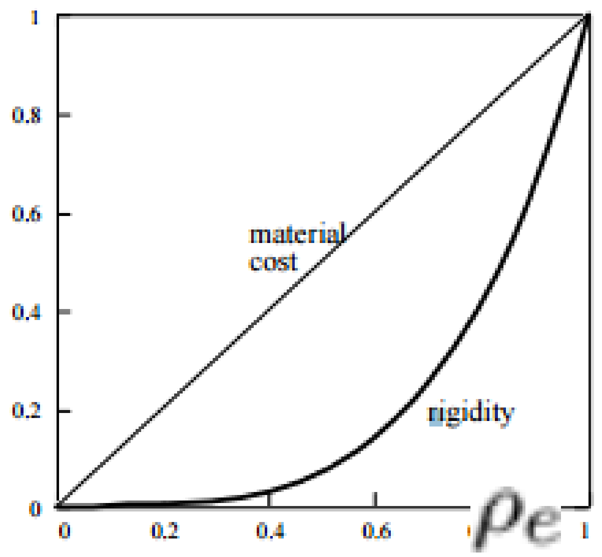
$$\rho_i = \begin{cases} 1 & \text{(solid)} \\ 0 & \text{(void)} \end{cases}, \forall i \quad \Rightarrow \quad \rho_i \in [0\,,1]$$

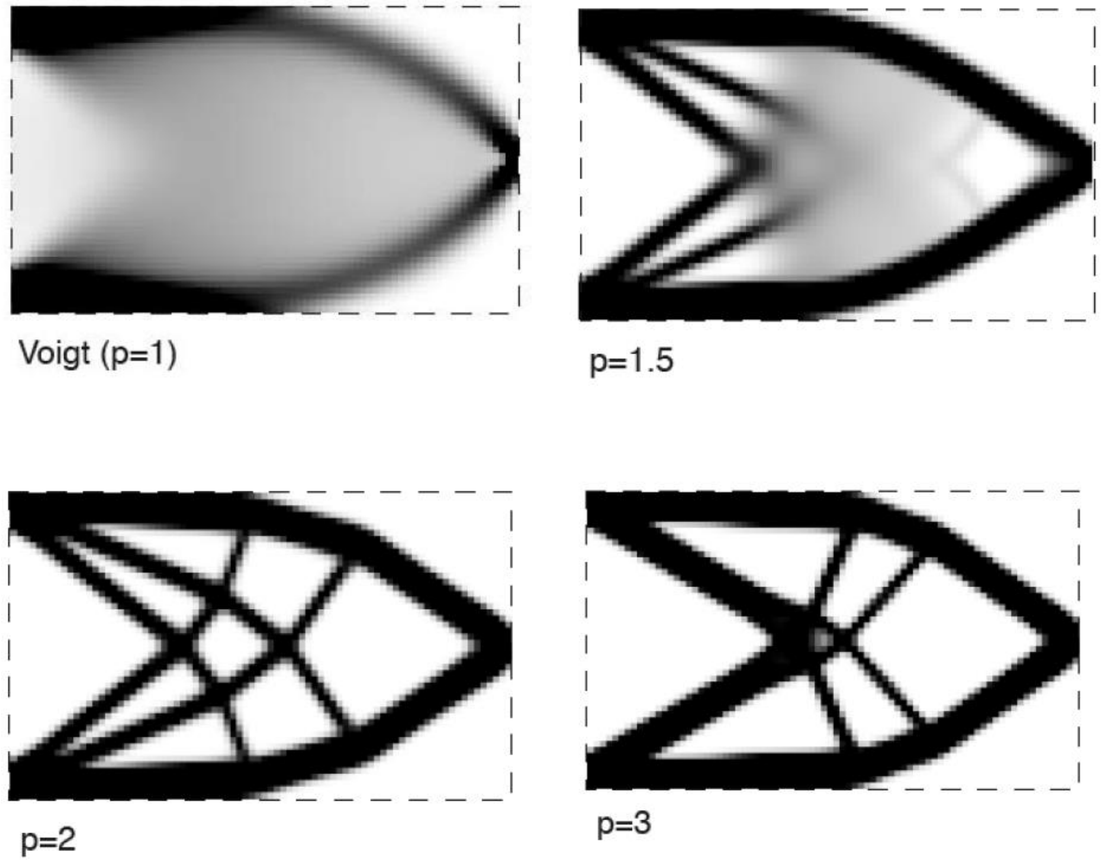
$$\mathrm{g} = \sum_i \rho_i - V_0 \leq 0$$

- Motivation: (Difficult) binary problem → (easier) continuous problem

Material Interpolation

- Material properties: Young's modulus E, and Poisson's ratio ν
- SIMP interpolation (Solid Isotropic Material with Penalization)
  - $E_i = {\rho_i}^p \bar{E}$
  - $p \geq 1$, typically $p = 3$





Sensitivity Analysis

- Sensitivity: The derivative of a function with respect to design variables
- $\frac{\partial c}{\partial \rho_i} = -\frac{p}{2}\rho_i^{p-1} u_i^T \overline{K} u_i$
  - Smaller than zero
- $\frac{\partial g}{\partial \rho_i} = 1$

Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

$\rho_i \in [0\,,1]$

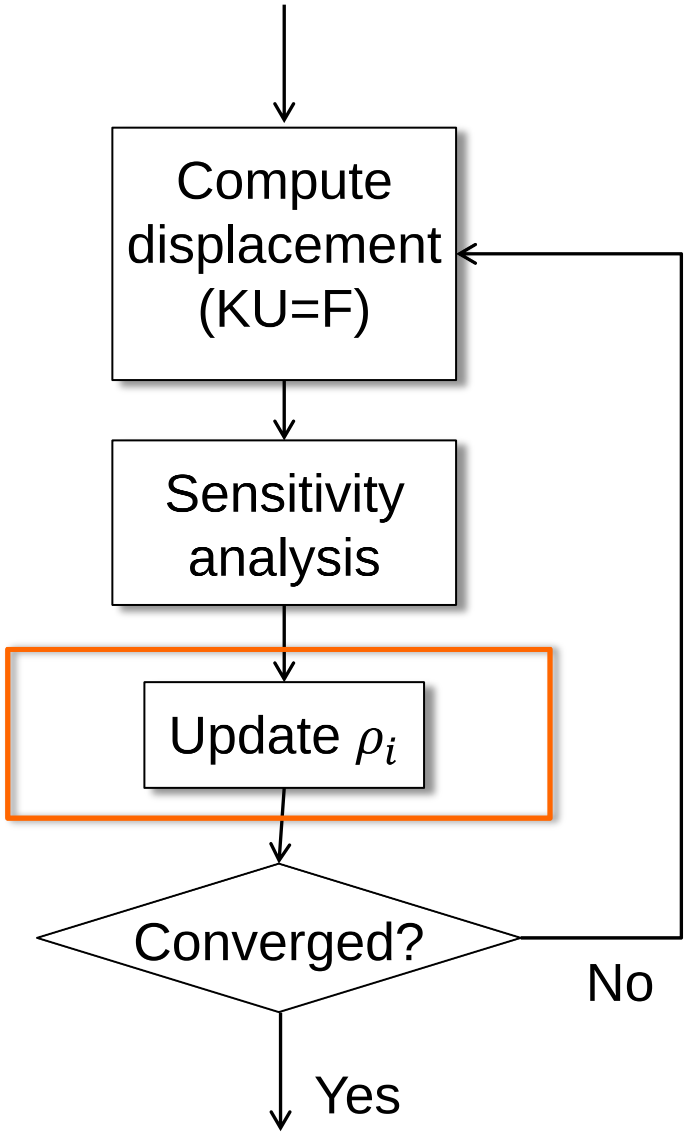
$\mathrm{g} = \sum_i \rho_i - V_0 \leq 0$

Design Update

- Mathematical programming
  - Interior point method (IPOPT package)
  - The method of moving asymptotes (MMA)

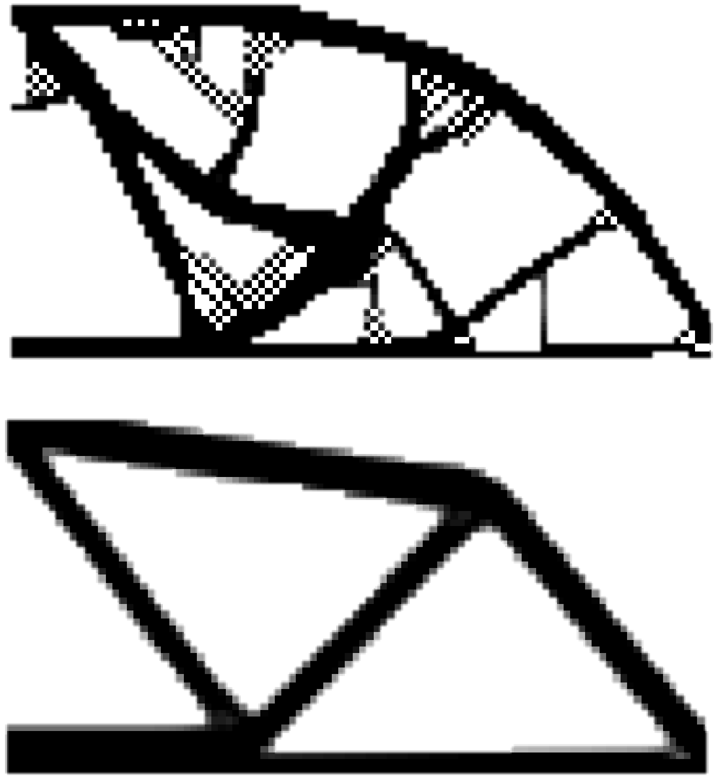
- Optimality criterion
  - If "$-\frac{\partial c}{\partial \rho_i}$" is large, increase ρ_i
  - Otherwise, decrease ρ_i

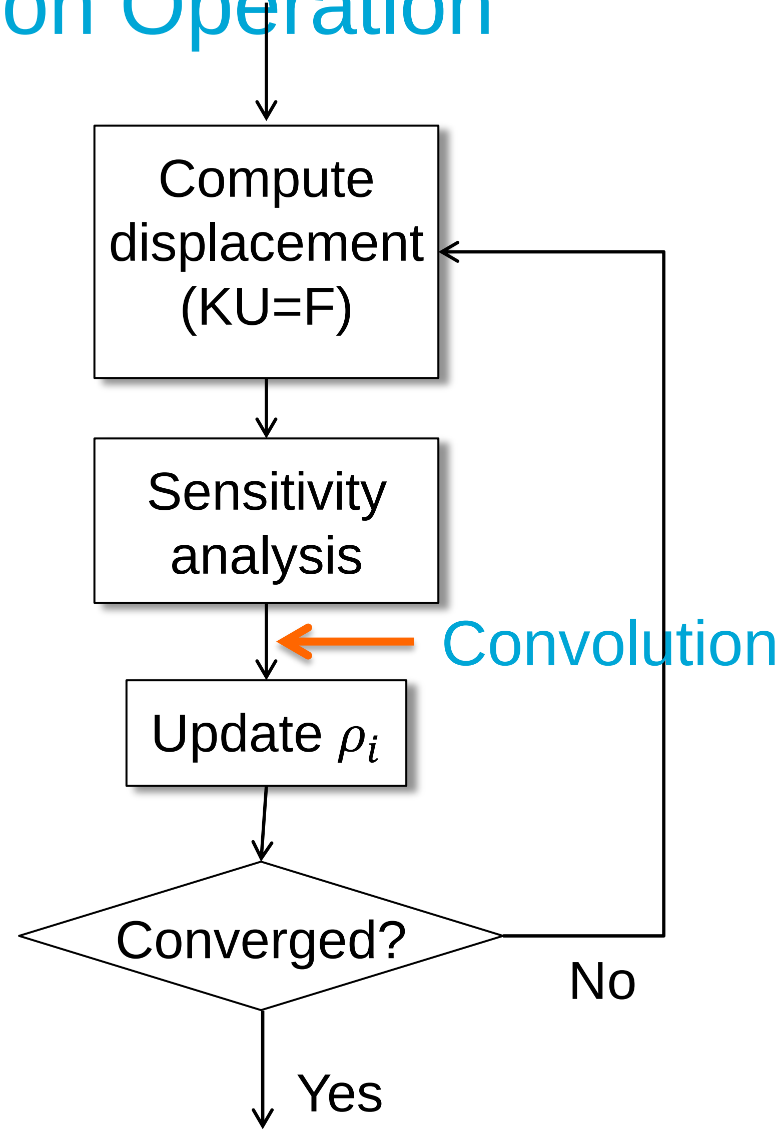
  - How to determine large or small?
  - Bisection search for a threshold



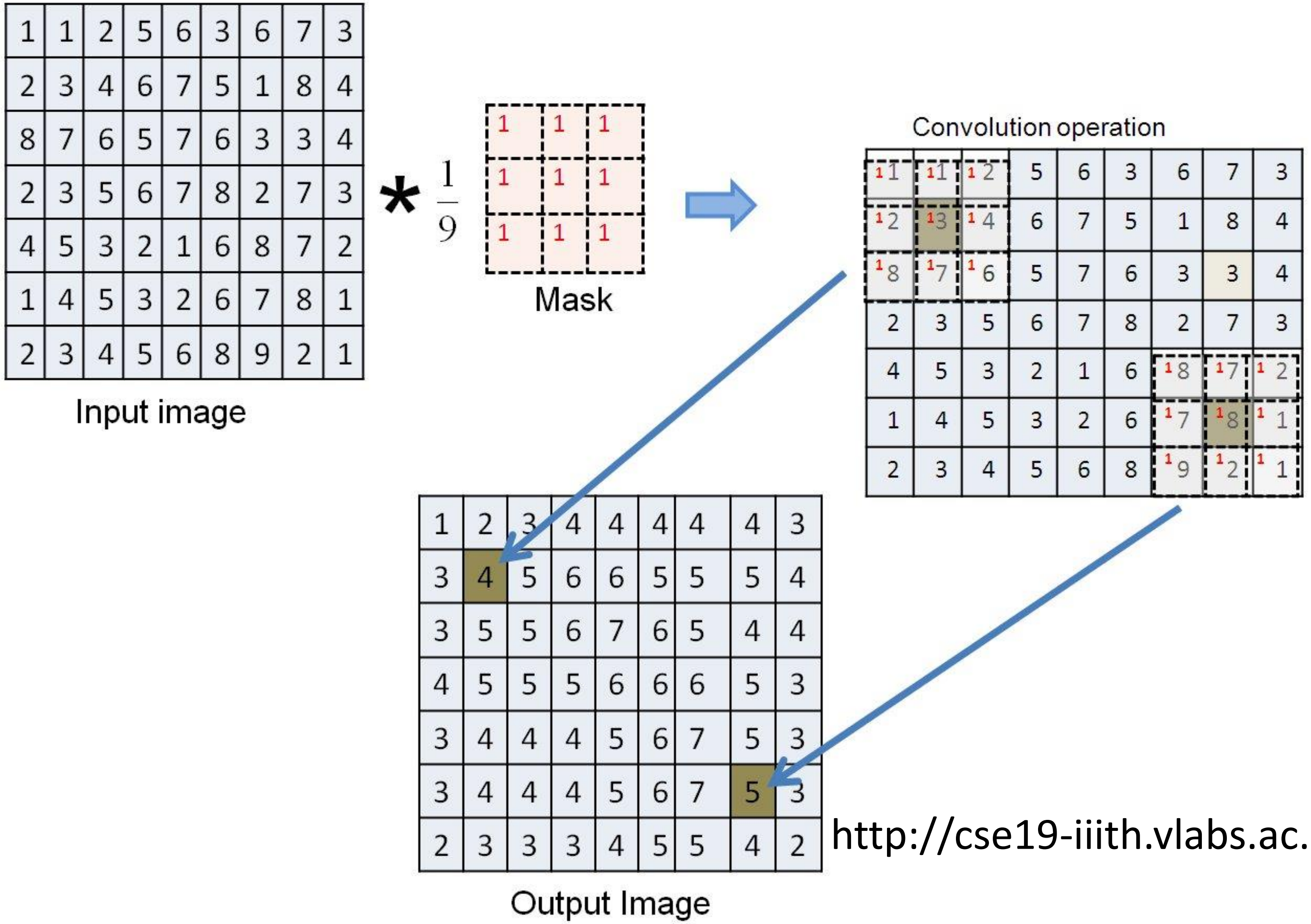
Checkerboard Patterns

Sensitivity Filtering by a Convolution Operation





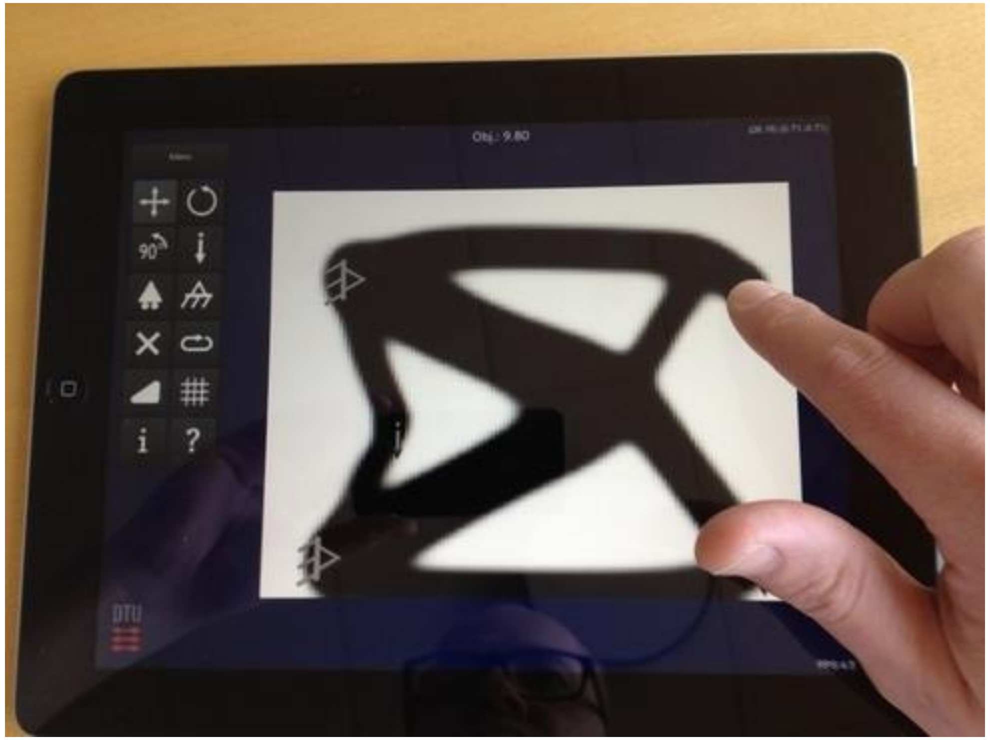
Convolution Operation



http://cse19-iiith.vlabs.ac.in/theory.php?exp=neigh

Demo

- www.topopt.dtu.dk



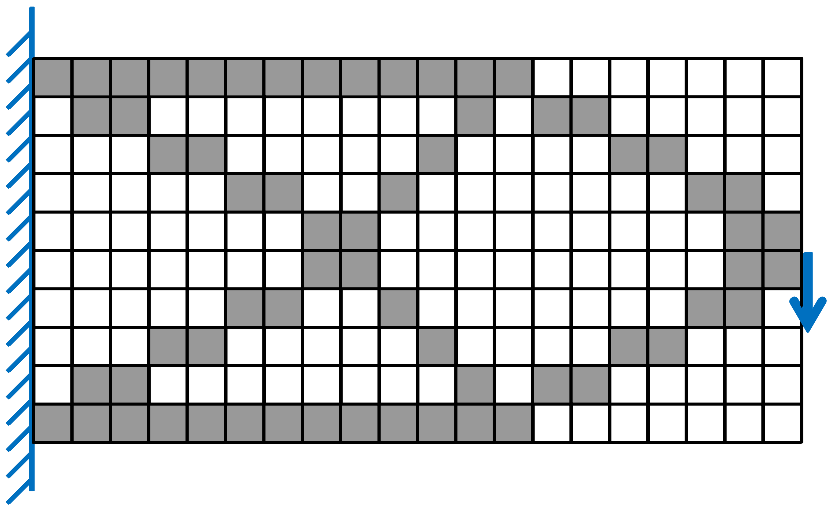
Topology Optimization

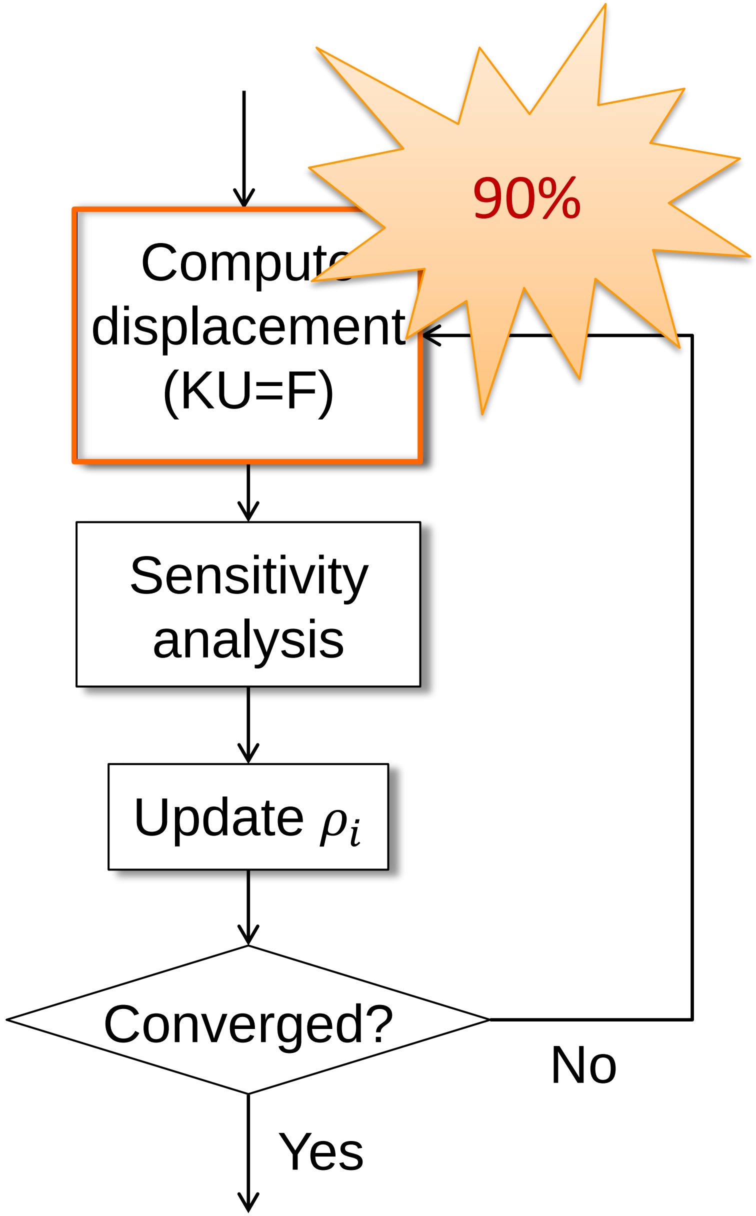
Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

$\rho_i \in [0,1], \forall i$

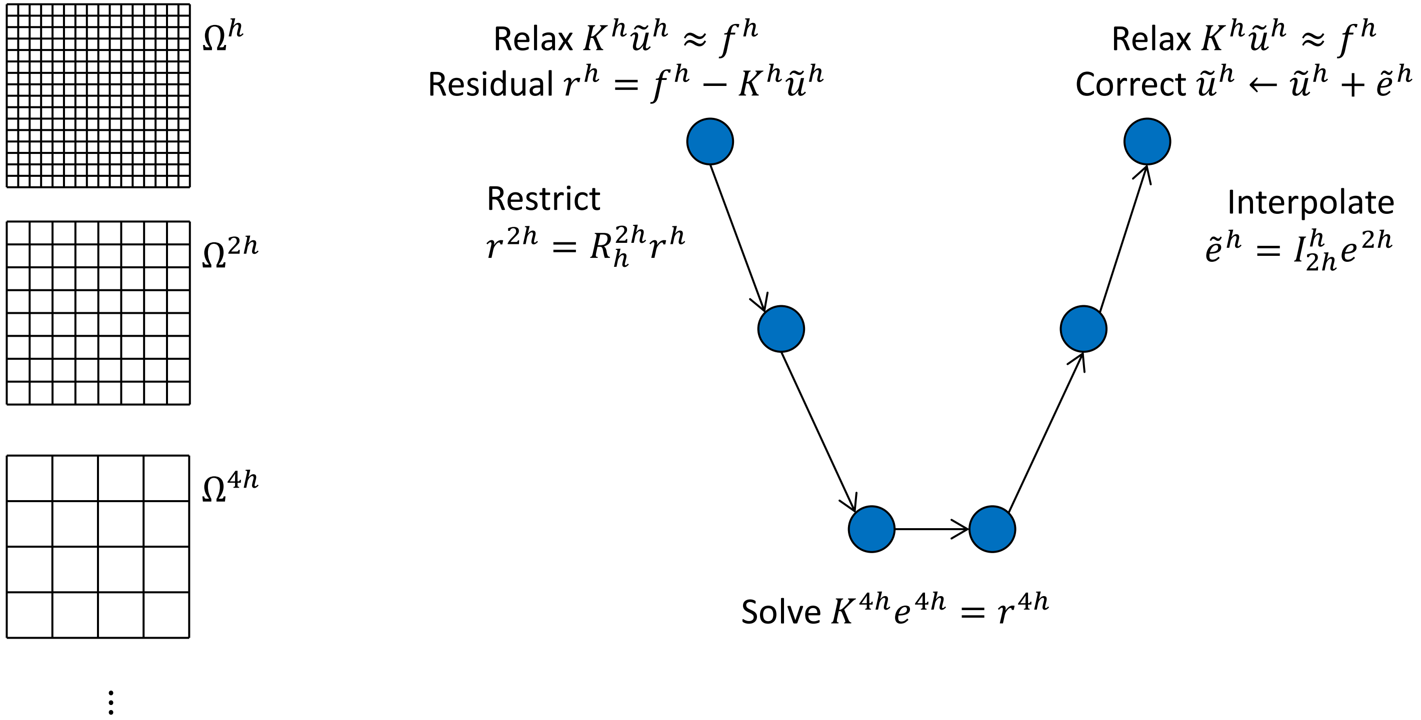
$g = \sum_i \rho_i - V_0 \leq 0$





Geometric Multigrid: Solving $Ku = f$

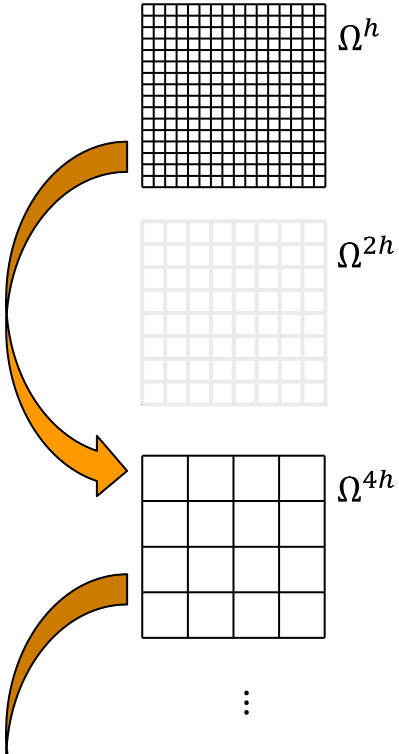
- Successively compute approximations u_m to the solution $\mathrm{u} = \lim_{m \to \infty} u_m$
- Consider the problem on a hierarchy of successively coarser grids to accelerate convergence

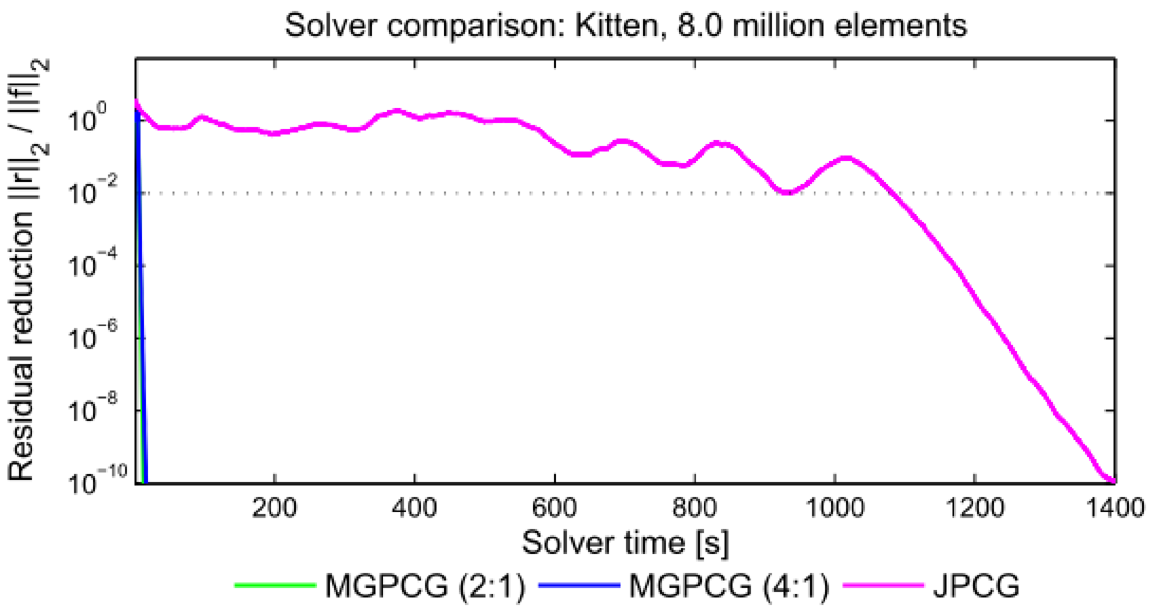


W. Briggs, A multigrid tutorial, 2000

Memory-Efficient Implementation on GPU

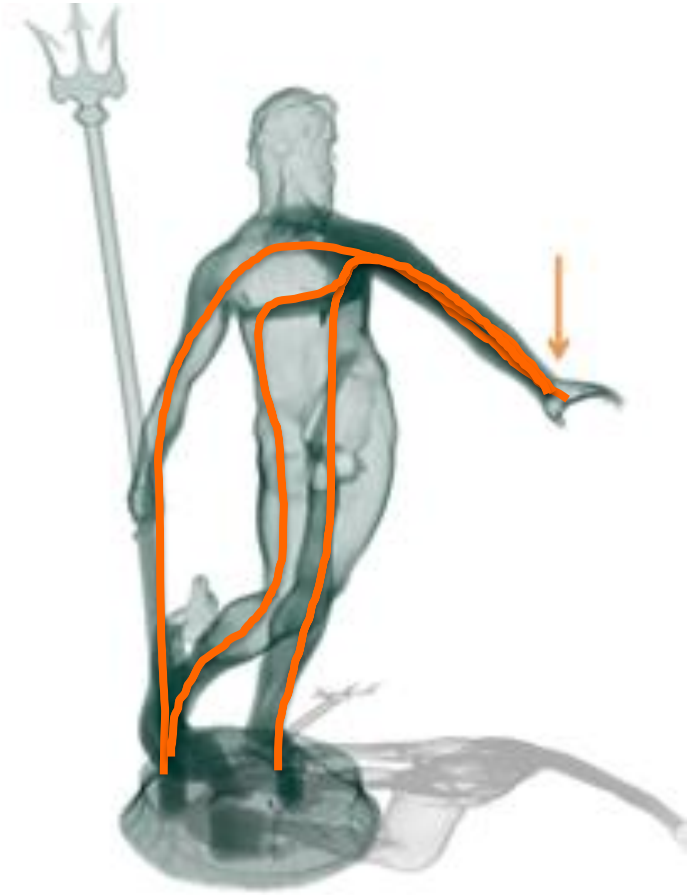
- On-the-fly assembly
  - Avoid storing matrices on the finest level
- Non-dyadic coarsening (i.e., 4:1 as opposed to 2:1)
  - Avoid storing matrices on the second finest level





Wu et al., TVCG'2016
Dick et al., SMPT'2011

High-Resolution Design



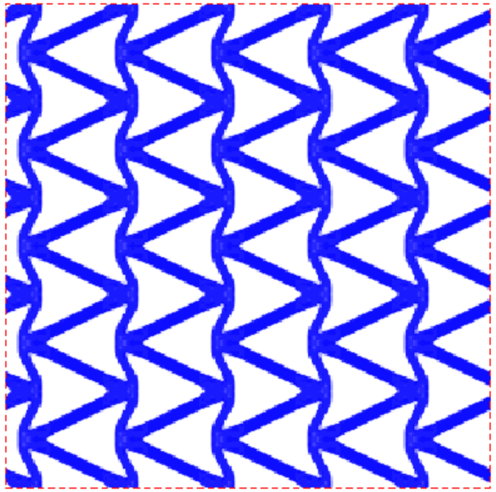
Resolution: 621 × 400 × 1000
#Element 14.2m
Time: 12 minutes

Kitten

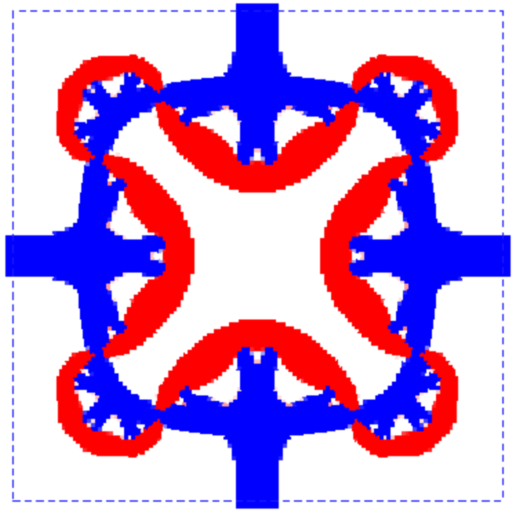
Resolution: **262 × 238 × 400**

Elements: **8 million**

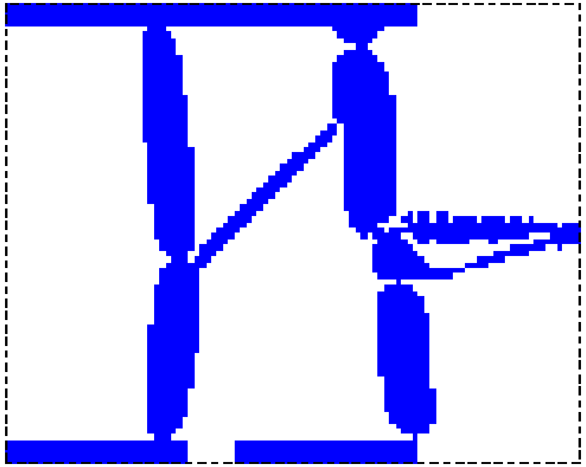
Target volume reduction: **60%**



Negative Poisson's ratio
Larsen et al. 1997



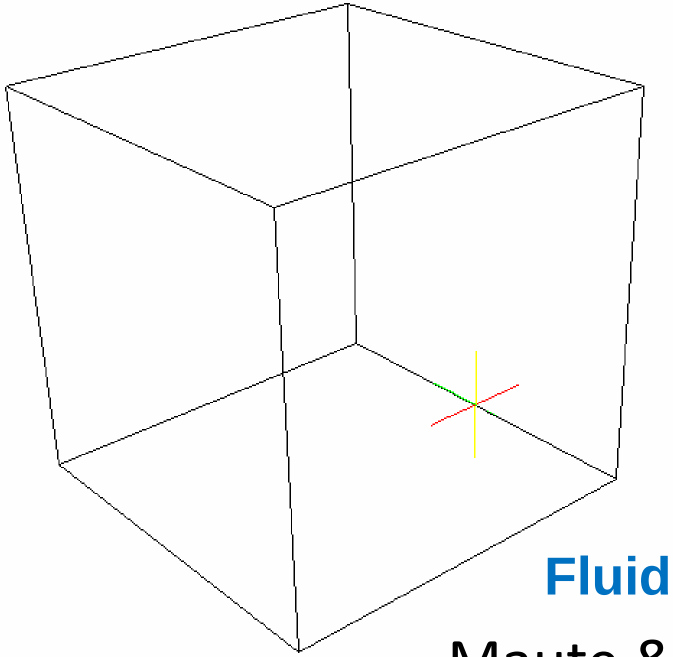
Negative thermal expansion
Sigmund &Torquato 1996



Electric actuator
Sigmund 2000

Natural convection

Alexandersen et al. 2016



Fluid flow

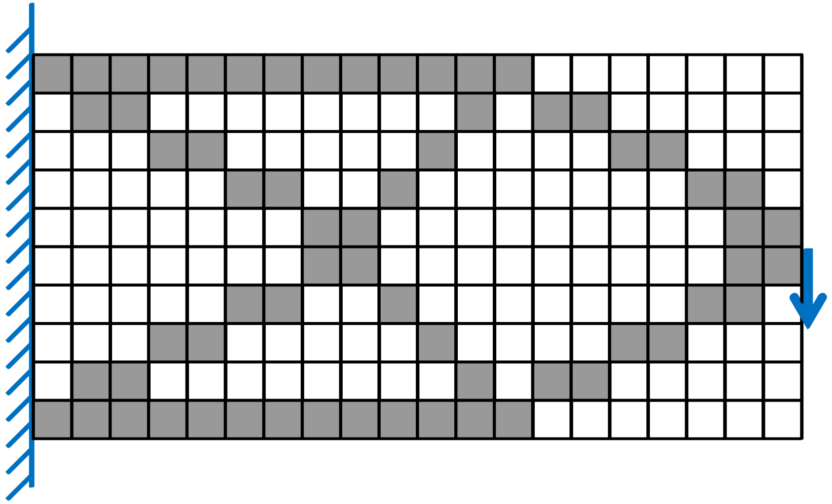
Maute & Pingen

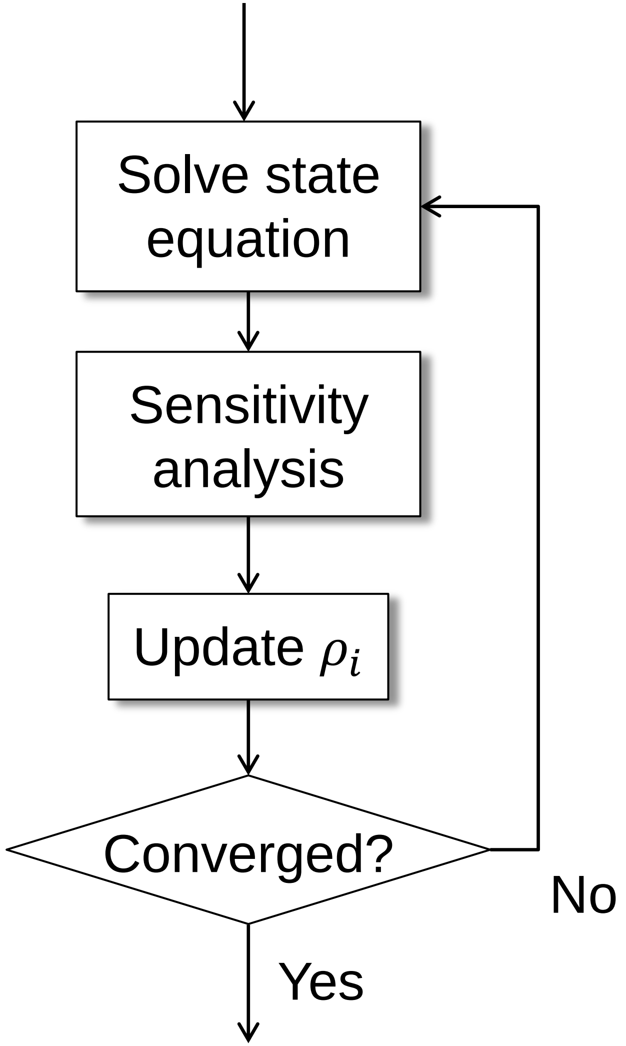
A General Formulation

Minimize: $c(\rho)$

Subject to: $\rho_i \in [0,1], \forall i$

$g_i(\rho) \leq 0$



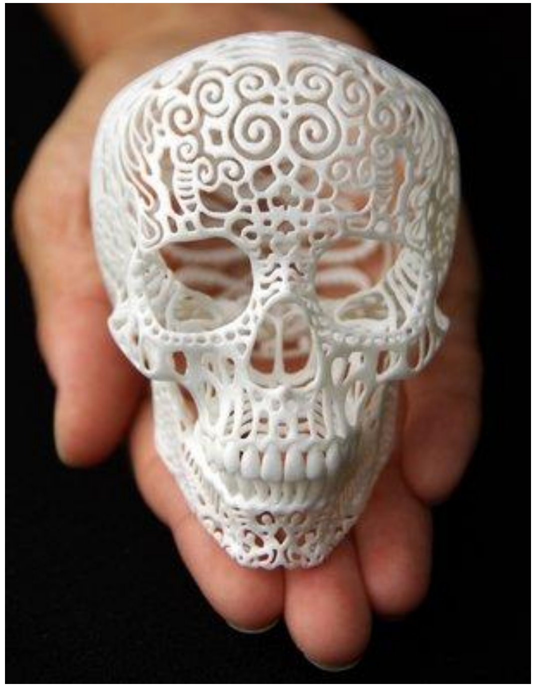


Outline

- Basics of Topology Optimization
- Topology Optimization for Additive Manufacturing

Additive Manufacturing: Complexity is free

TU Delft & MX3D, 2015



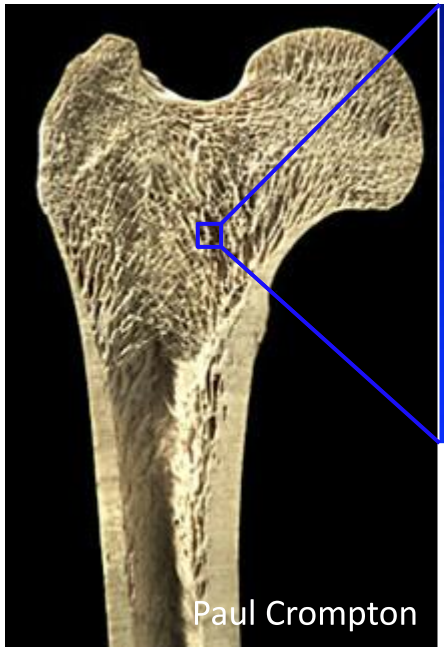
Joshua Harker

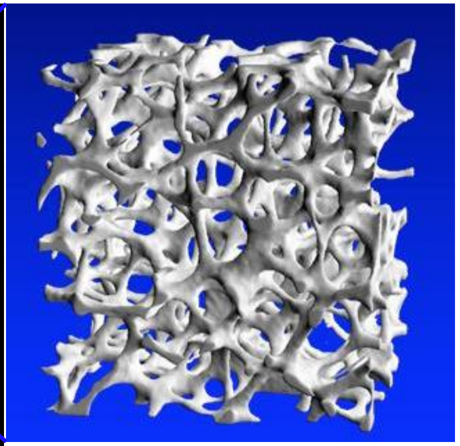
Scott Summit

Complexity is free? … Not really!

- Printer resolution: Minimum geometric feature size
- Layer-upon-layer: Supports for overhang region
- Shell-infill composite

Tiny details



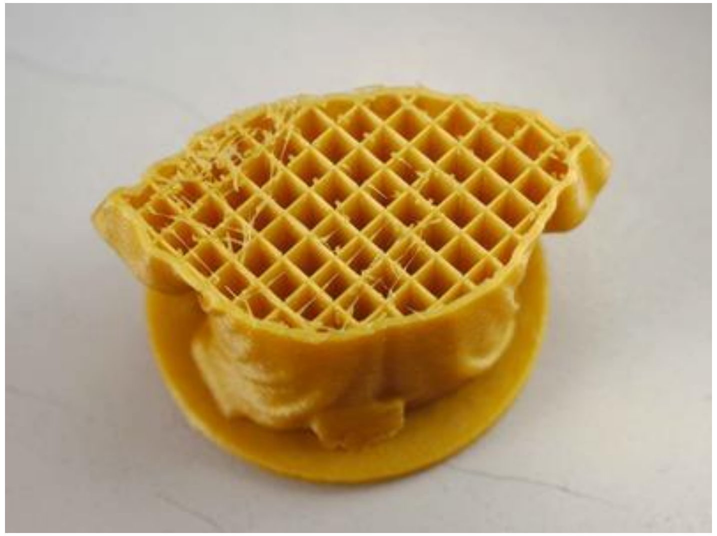


Ralph Müller

Supports

Concept Laser GmhH

Infill



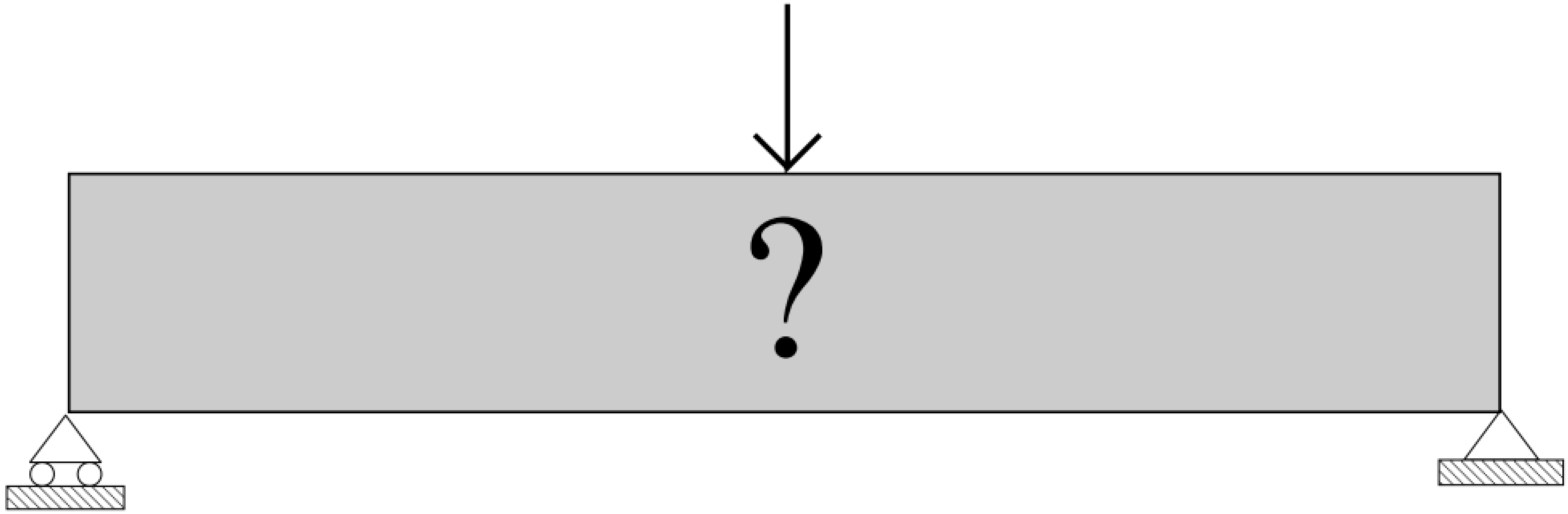
mpi.fs.tum.de

Outline

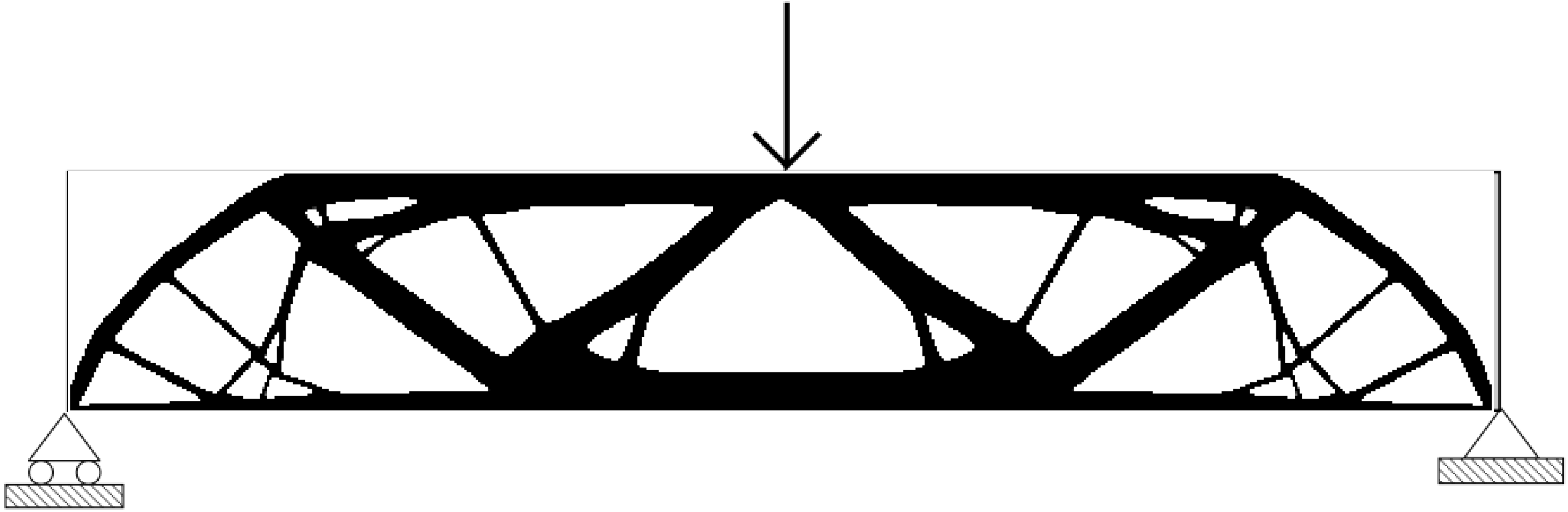
- Basics of Topology Optimization

- Topology Optimization for Additive Manufacturing
  - Geometric feature control by **density filters**
  - Geometric feature control by **alternative parameterizations**

Messerschmidt-Bölkow-Blohm (MBB) beam



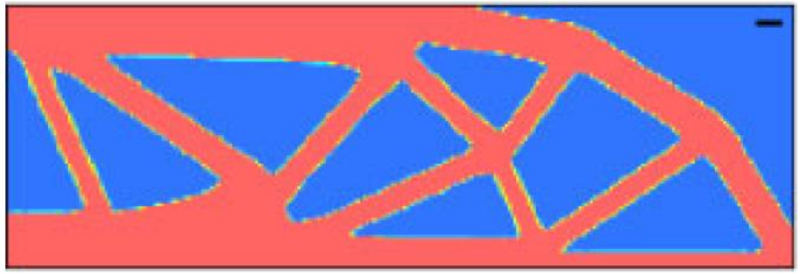
Messerschmidt-Bölkow-Blohm (MBB) beam



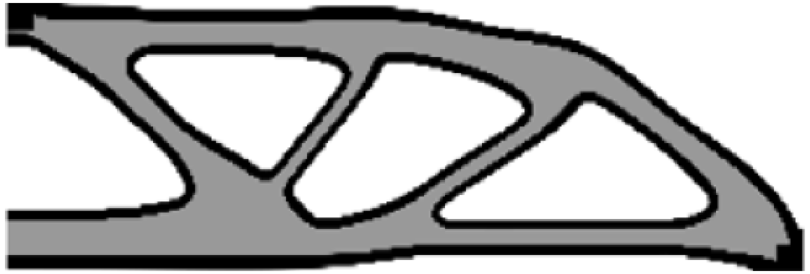
Geometric feature control by density filters
(An incomplete list)

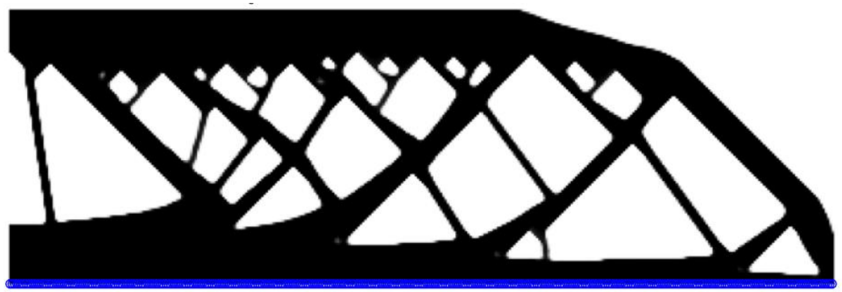
Reference

Minimum feature size, Guest'04

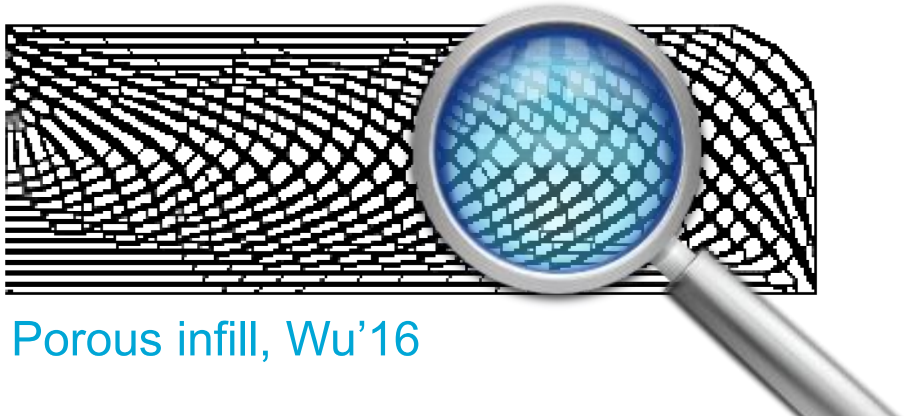


Coating structure, Clausen'15



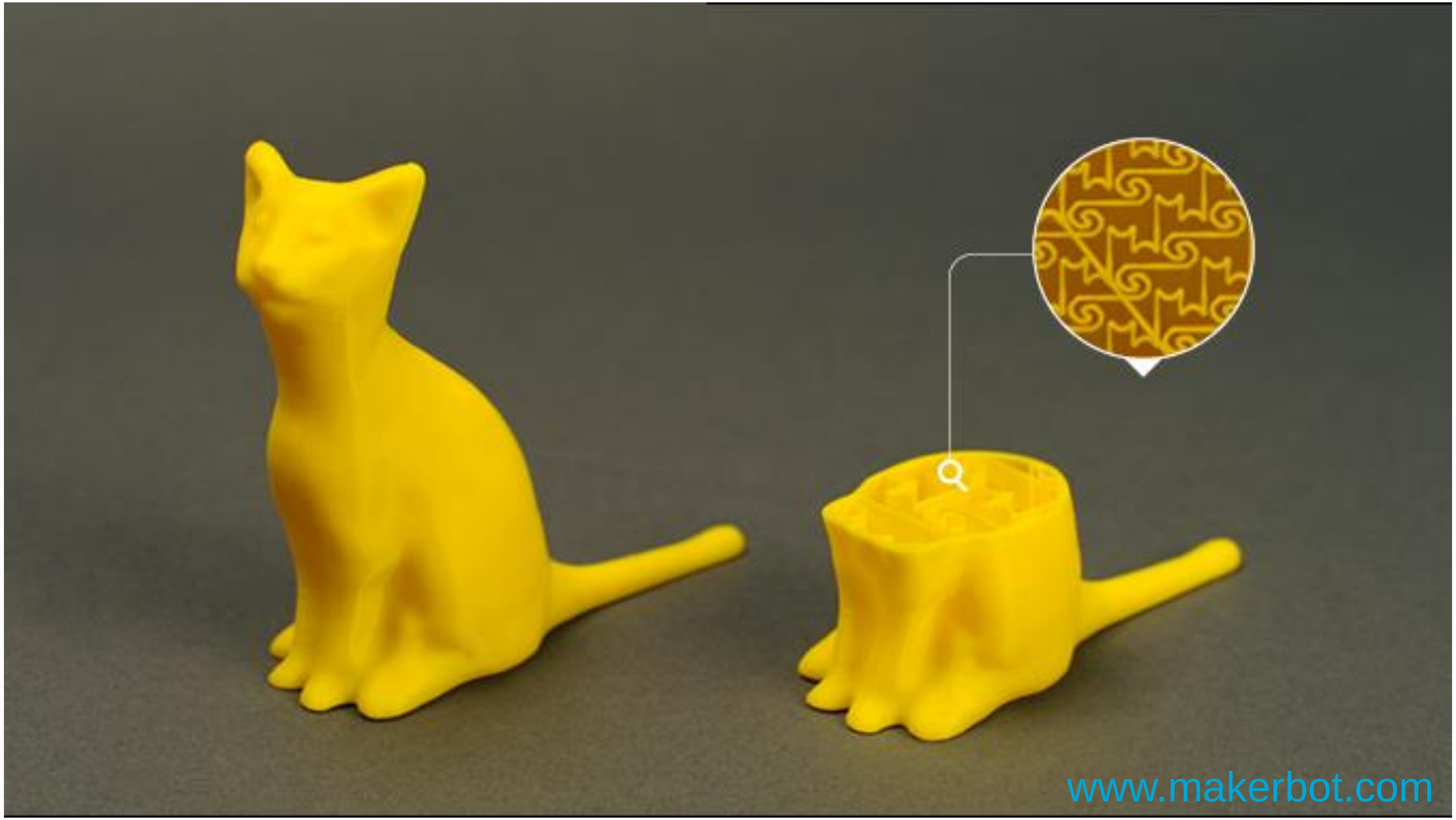


Self-supporting design, Langelaar'16



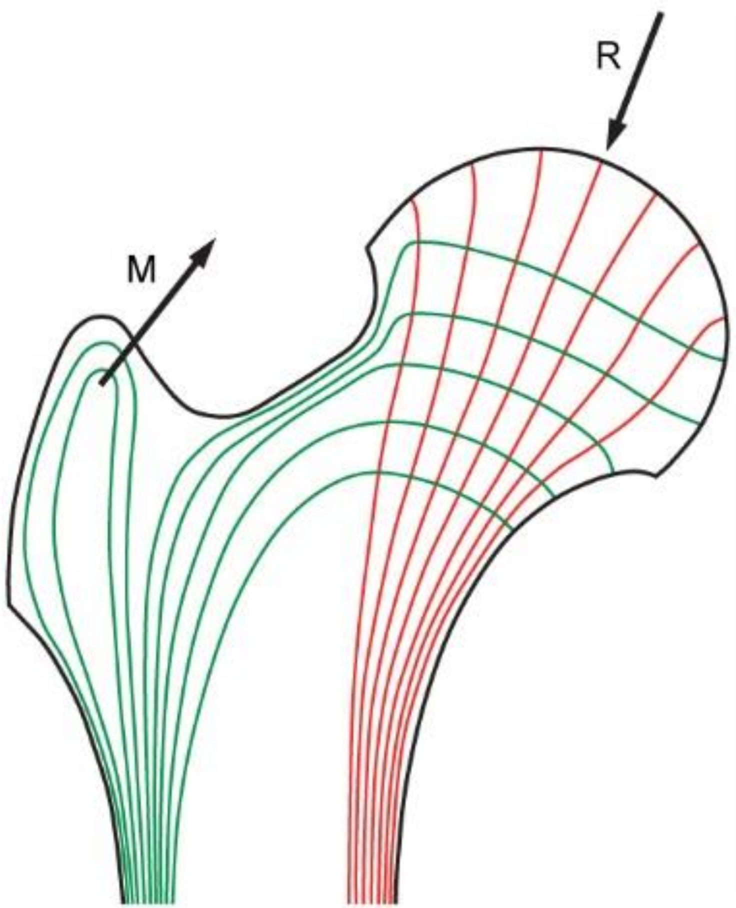
Porous infill, Wu'16

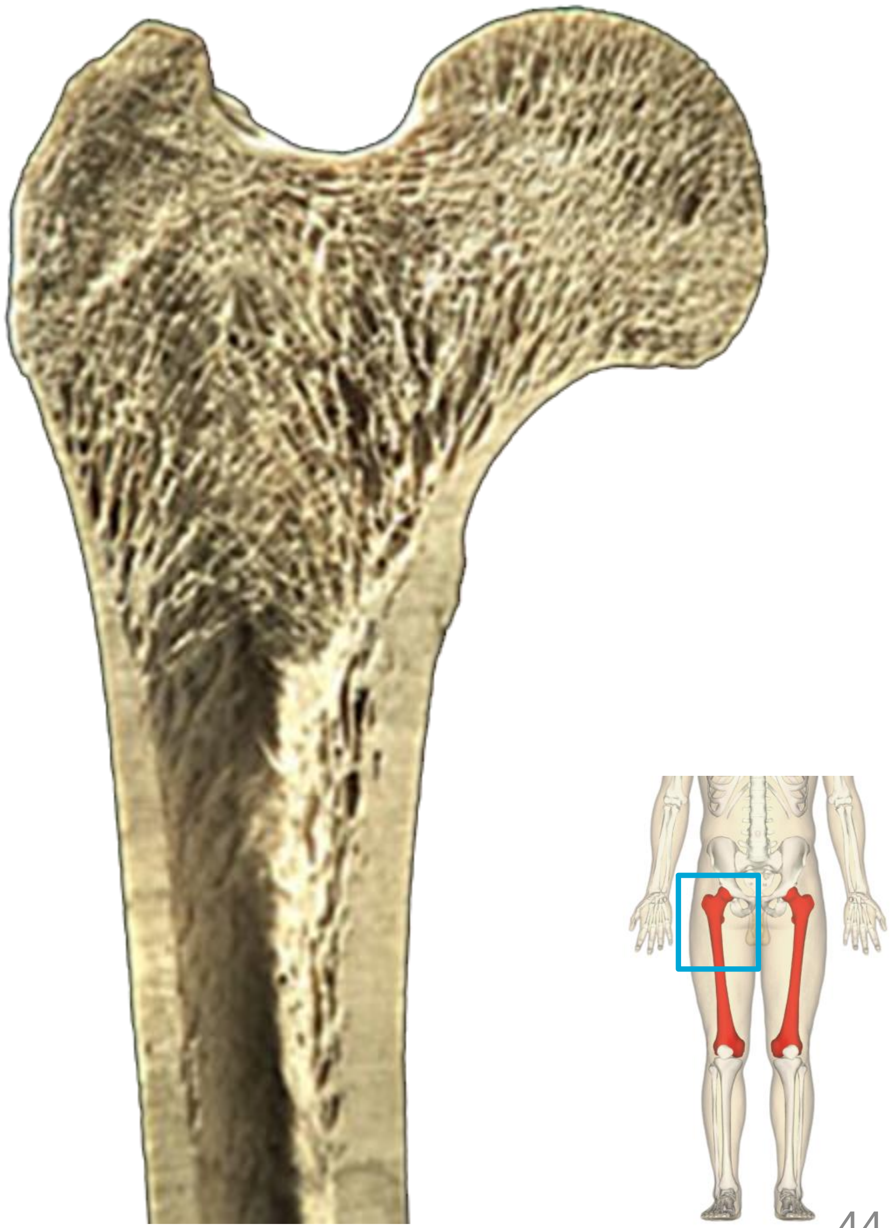
Infill in 3D Printing: Regular Structures



3dplatform.com

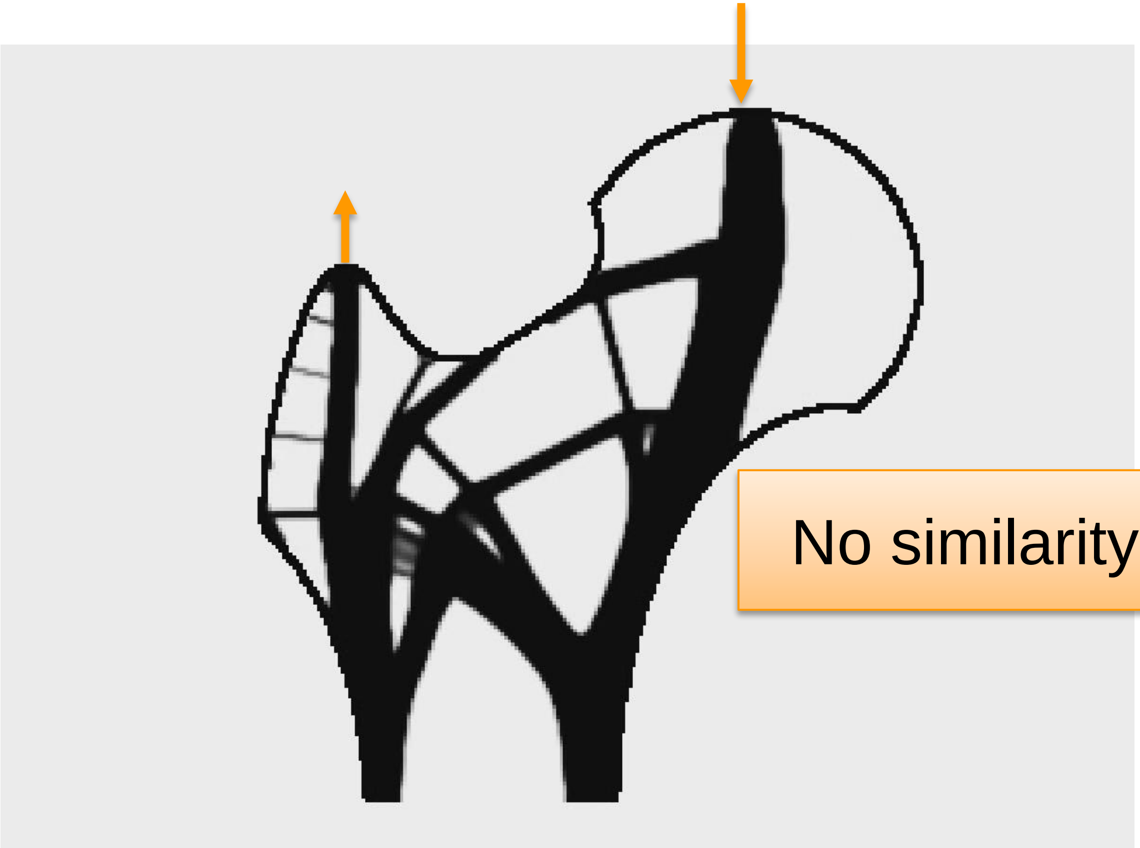
Infill in Bone: Porous Structures



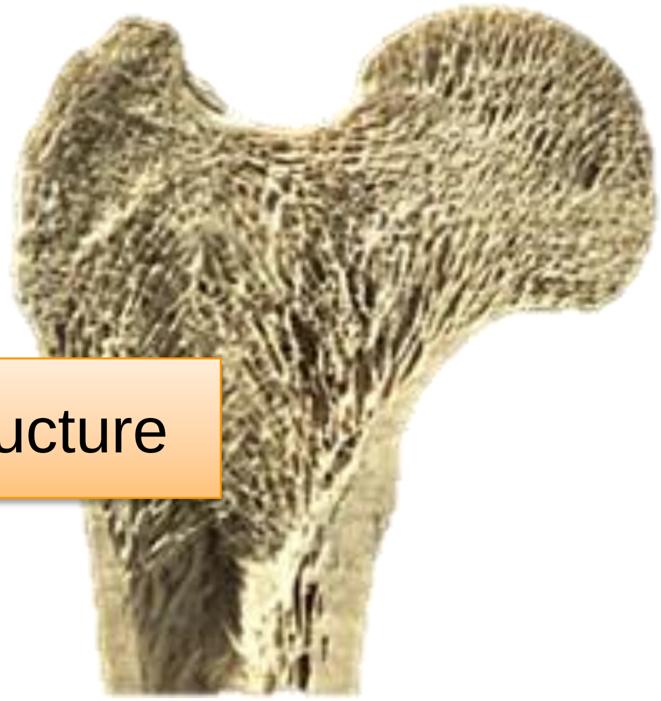


Can we apply the principle of bone to 3D printing?

Topology Optimization Applied to Design Infill



Topology optimization



Infill in the bone

No similarity in structure

Topology Optimization Applied to Design Infill

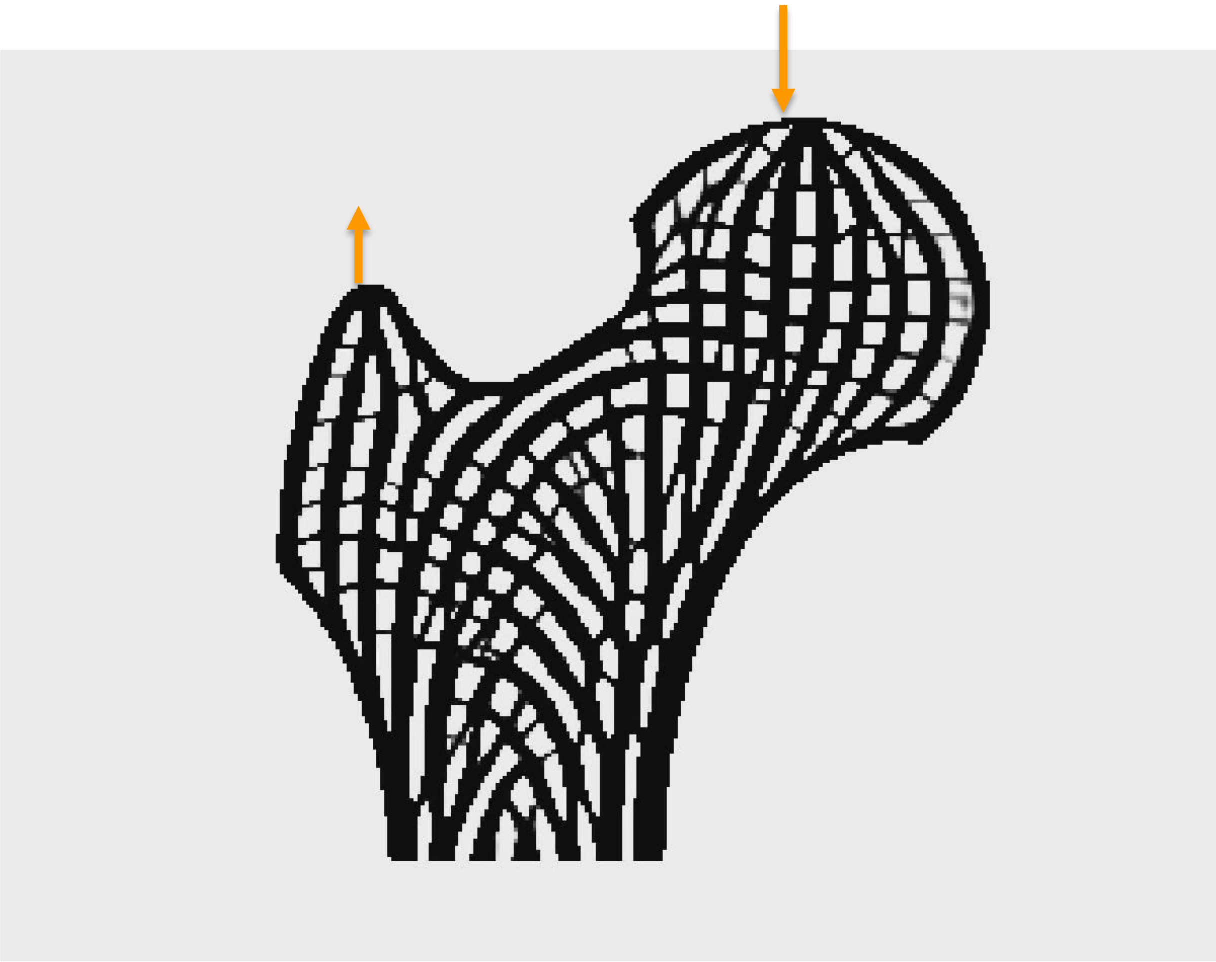
- Materials accumulate to “important” regions
- The total volume $\sum_i \rho_i v_i \leq V_0$ does not restrict local material distribution

Infill by standard topology optimization

Infill in the bone

Bone-like Infill in 2D

Cross-section of a human femur



Approaching Bone-like Structures: The Idea

- Impose **local constraints** to avoid fully solid regions

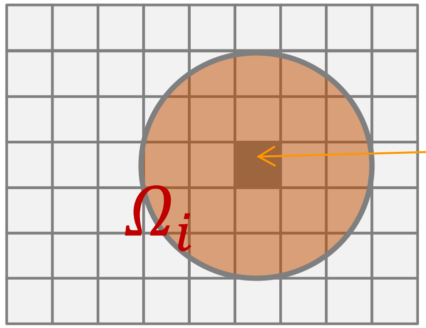
Min: $c = \frac{1}{2} U^T K U$

s.t. : $KU = F$

$\rho_i \in [0,1], \forall i$

~~$\sum_i \rho_i \leq V_0$~~

$\hat{\rho}_i \leq \alpha, \forall i$



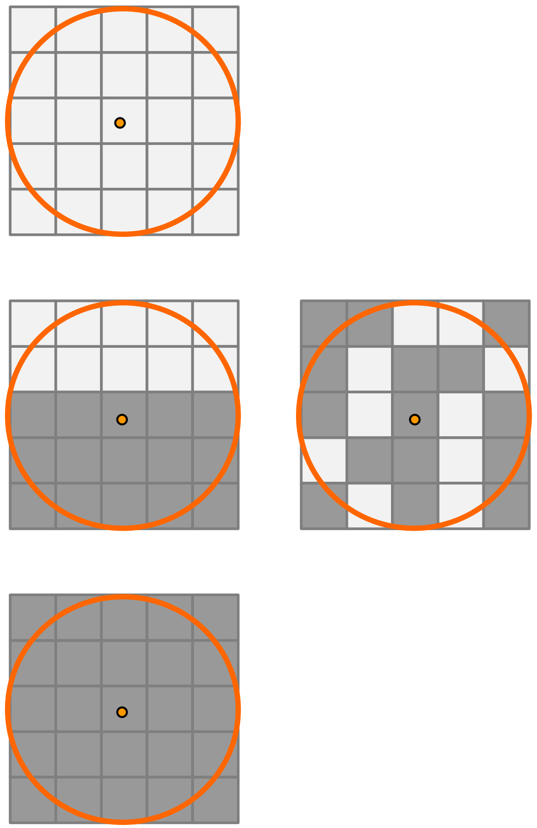
$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

$\hat{\rho}_i = 0.0$

$\hat{\rho}_i = 0.6$

$\hat{\rho}_i = 1.0$



Constraints Aggregation (Reduce the Number of Constraints)

$\hat{\rho}_i \leq \alpha, \forall i$ ⟺ $\max_{i=1,\ldots,n} |\hat{\rho}_i| \leq \alpha$ ⟺ $\lim_{p\to\infty} \|\rho\|_p = \left(\sum_i (\hat{\rho}_i)^p\right)^{\frac{1}{p}} \leq \alpha$

Too many constraints!

A single constraint
But non-differentiable

A single constraint
and differentiable
Approximated with $p = 16$

Optimization Process: The same as in the standard topopt

- Impose **local constraints** to avoid fully solid regions

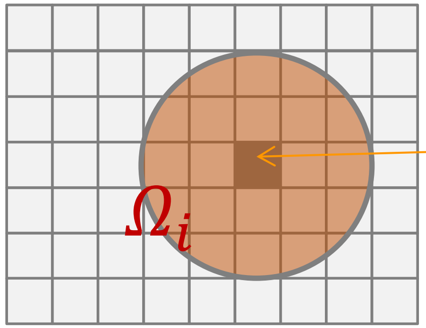
Min: $c = \frac{1}{2} U^T K U$

s.t. : $KU = F$

$\rho_i \in [0,1], \forall i$

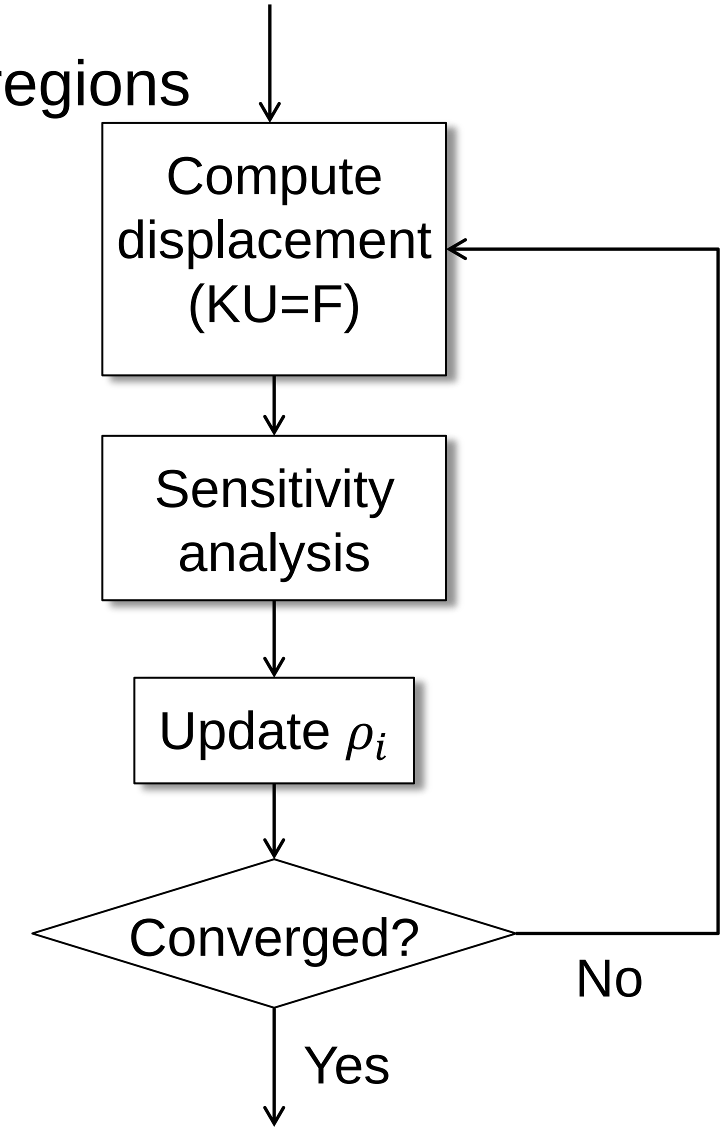
~~$\sum_i \rho_i \leq V_0$~~

$\hat{\rho}_i \leq \alpha, \forall i$

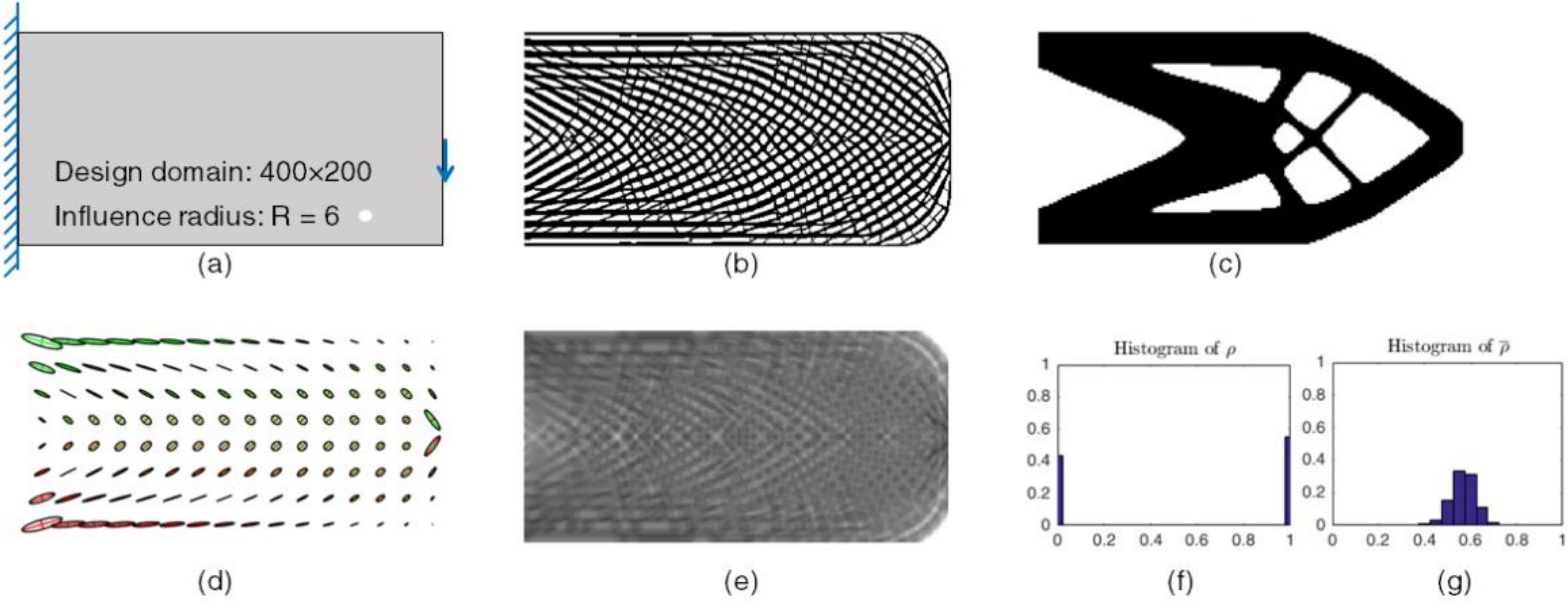


$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

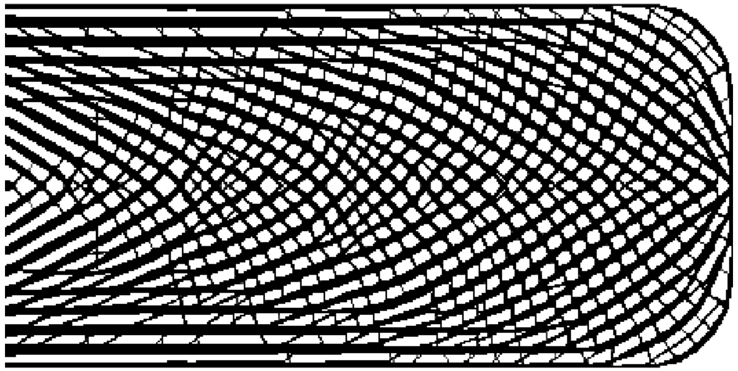


A Test Example

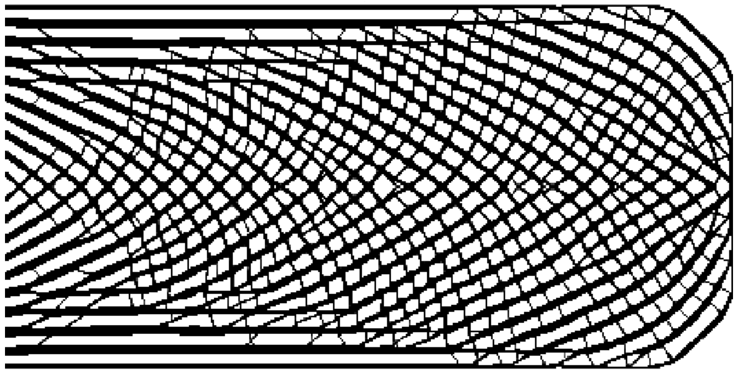


Effects of Filter Radius and Local Volume Upper Bound

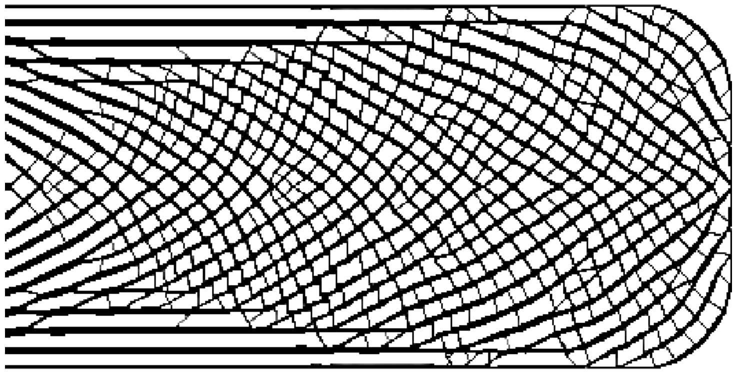
R=6



$(\alpha, c) = (0.6, 76.9)$

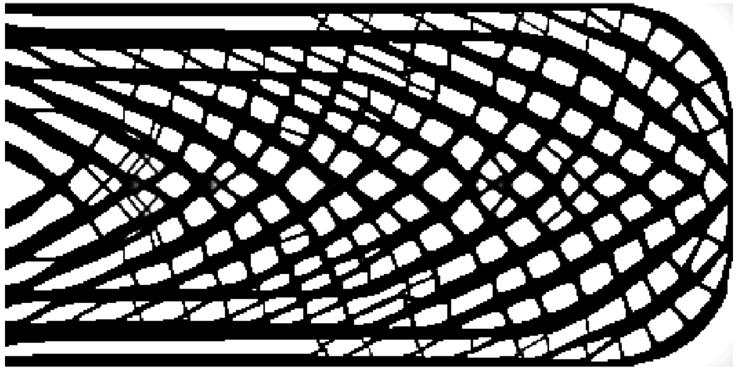


$(0.5, 96.0)$

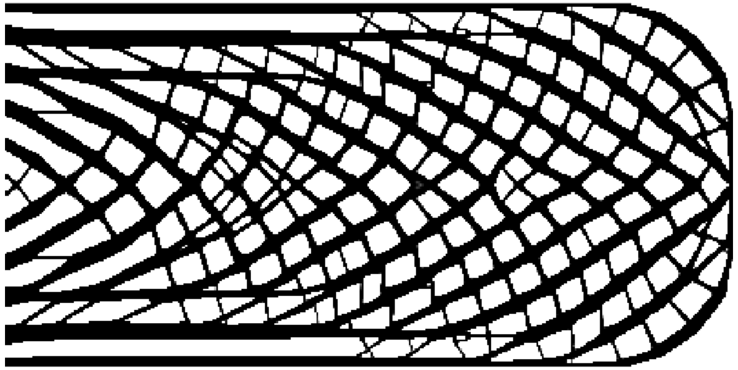


$(0.4, 130.0)$

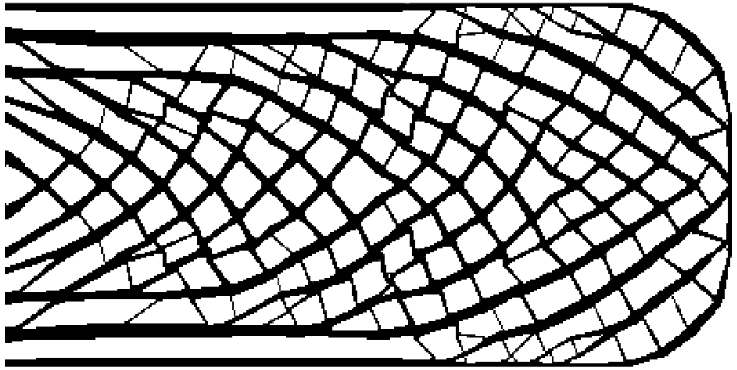
R=12



$(0.6, 73.9)$

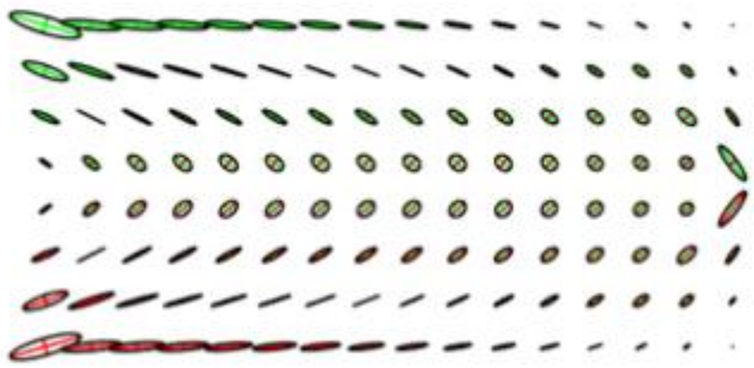


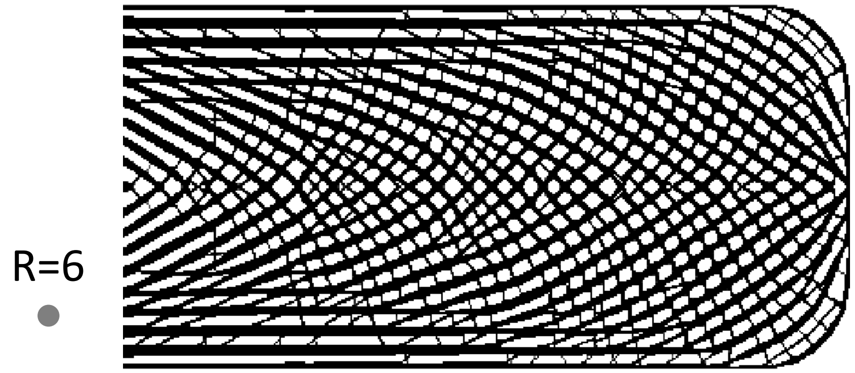
$(0.5, 91.2)$



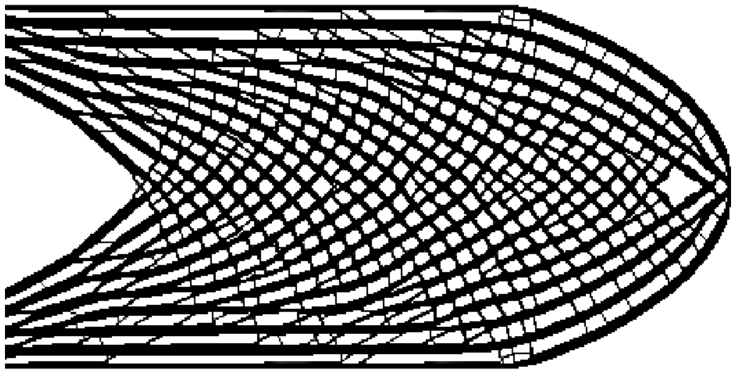
$(0.4, 119.8)$

Local + Global Volume Constraints

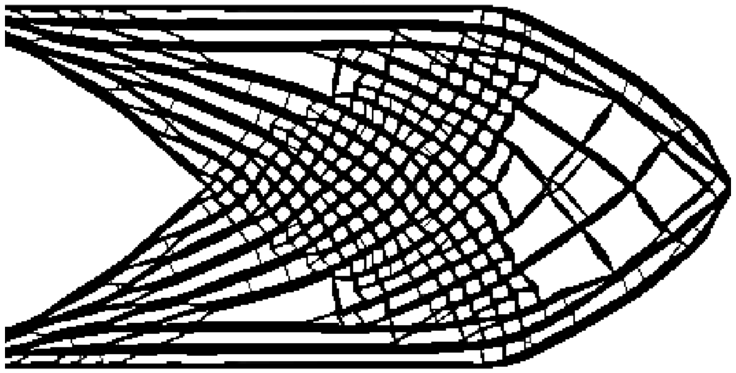




$(\alpha, \alpha_{total}, c) = (0.6, 0.56, 76.9)$

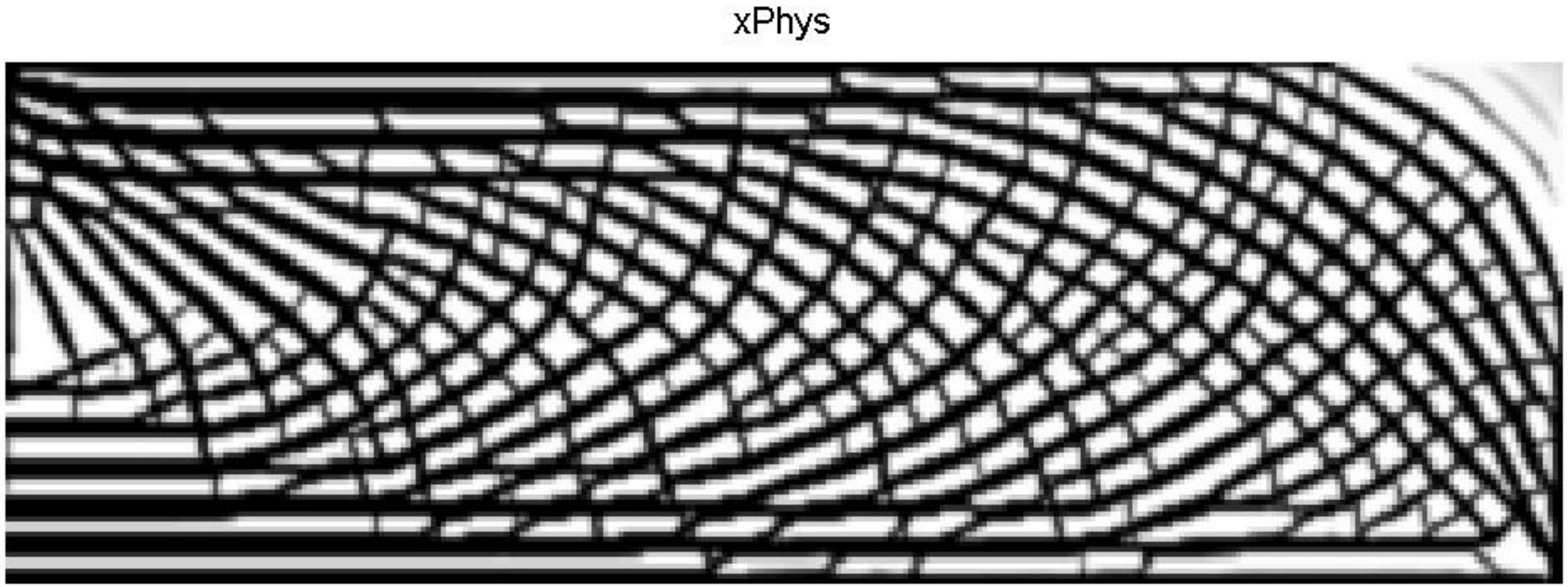


$(0.6, 0.50, 79.1)$

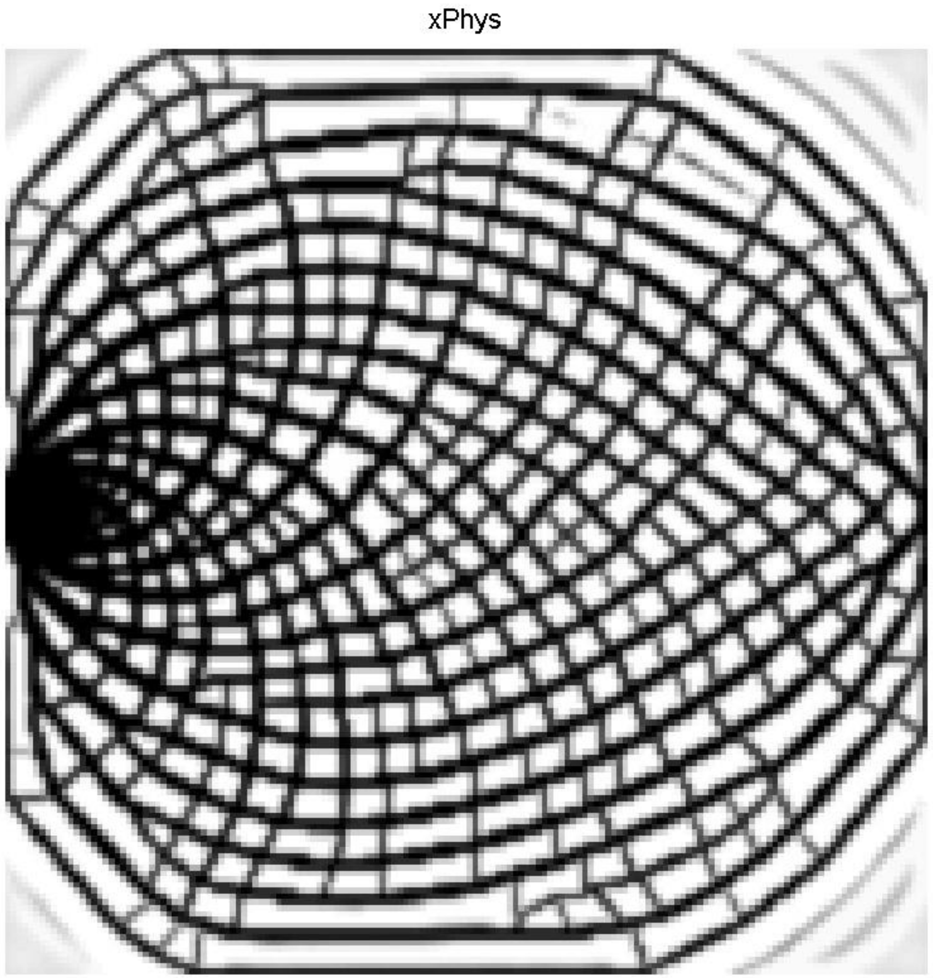


$(0.6, 0.40, 94.0)$

Result: 2D Animation



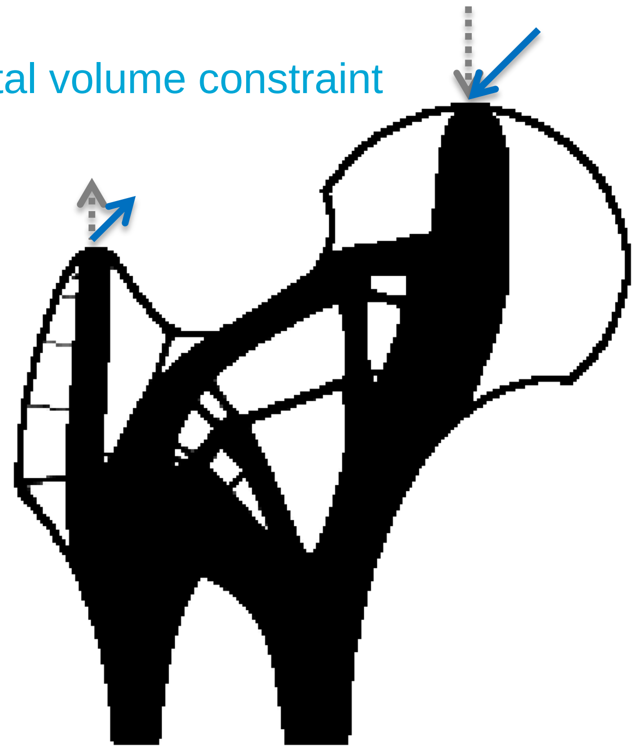
Result: 2D Animation



Robustness wrt. Force Variations

- Porous structures are significantly stiffer (126%) in case of force variations

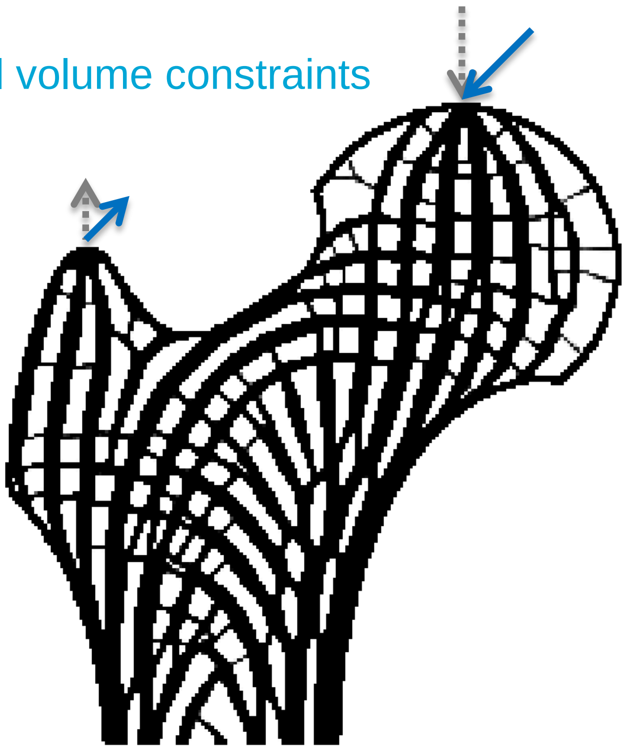
Total volume constraint



c = 30.54

c’= 45.83

Local volume constraints



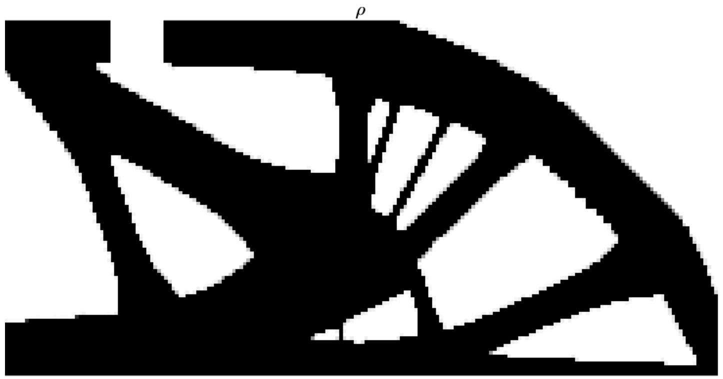
c = 36.72

c’ =36.23

Robustness wrt. Material Deficiency

- Porous structures are significantly stiffer (180%) in case of material deficiency

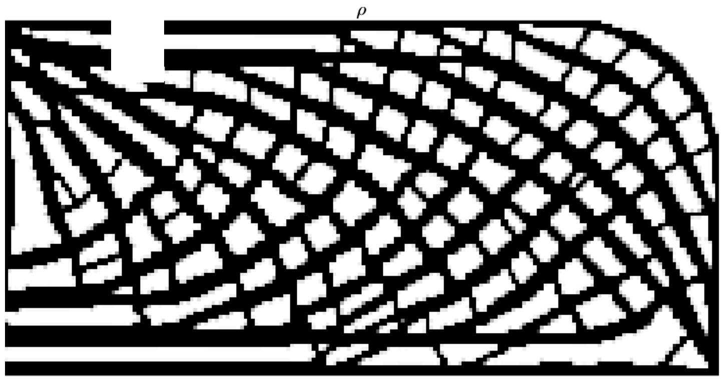
Total volume constraint



c = 76.83

c' =242.77

Local volume constraints

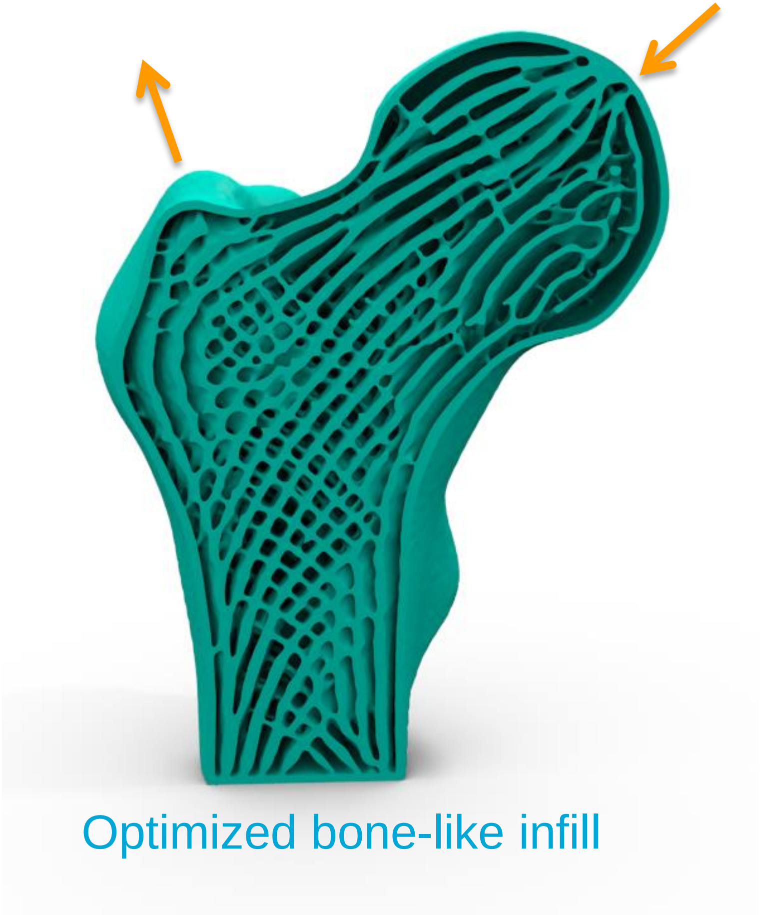


c = 93.48

c'= 134.84

Bone-like Infill in 3D

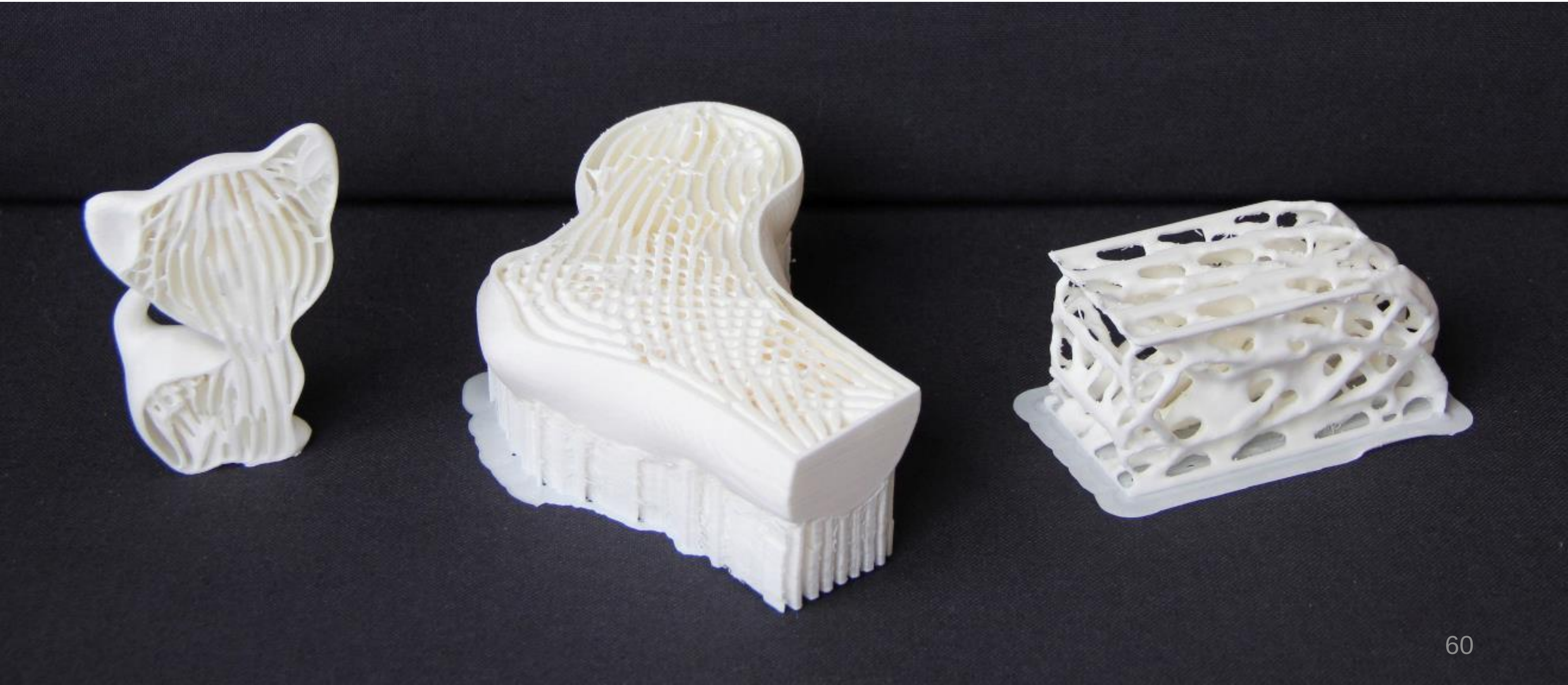
Infill in the bone



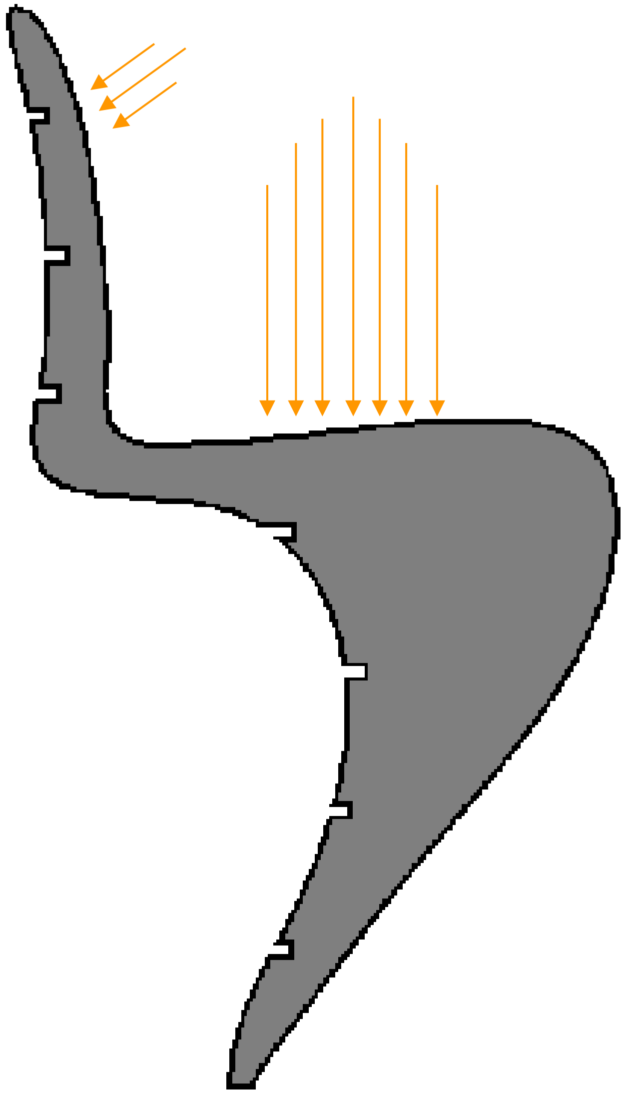
Optimized bone-like infill

Wu et al., TVCG'2017

FDM Prints



Chair

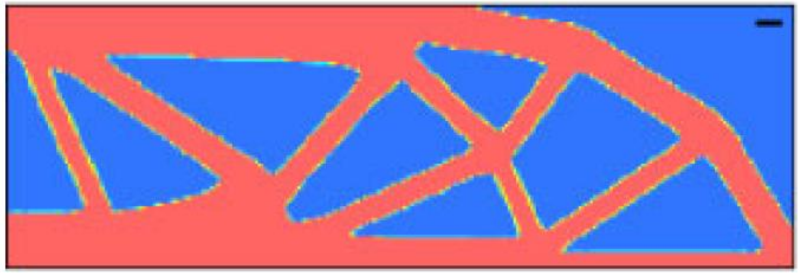



It’s what’s on the inside
that matters

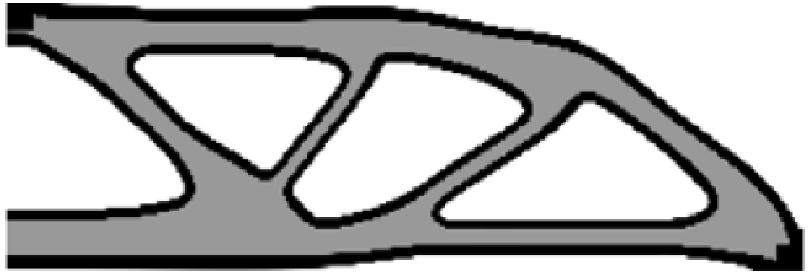
Geometric feature control by density filters
(An incomplete list)

Reference

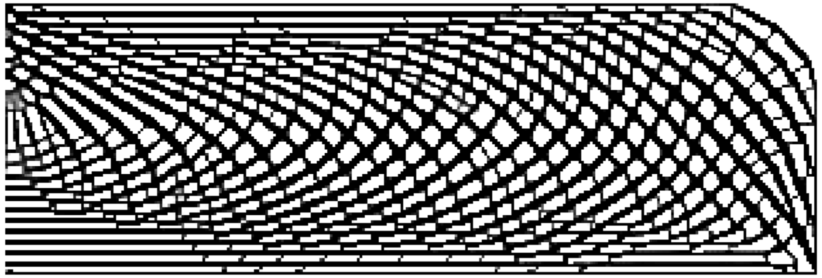
Minimum feature size, Guest'04



Coating structure, Clausen'15



Self-supporting design, Langelaar'16



Porous infill, Wu'16

Concurrent Shell-Infill Optimization

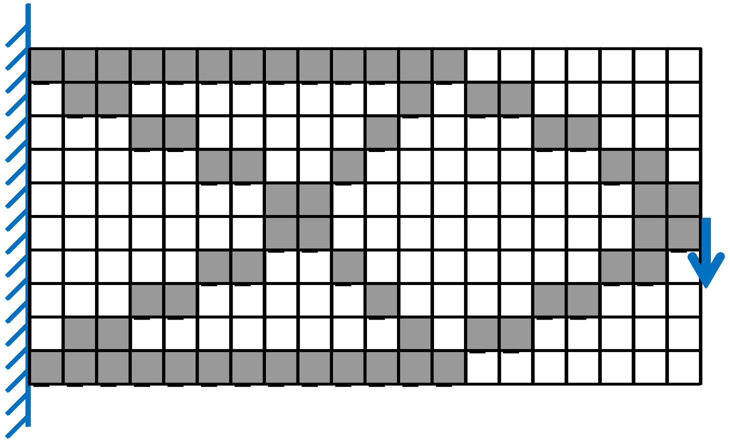
Wu et al., CMAME 2017

Outline

- Basics of Topology Optimization

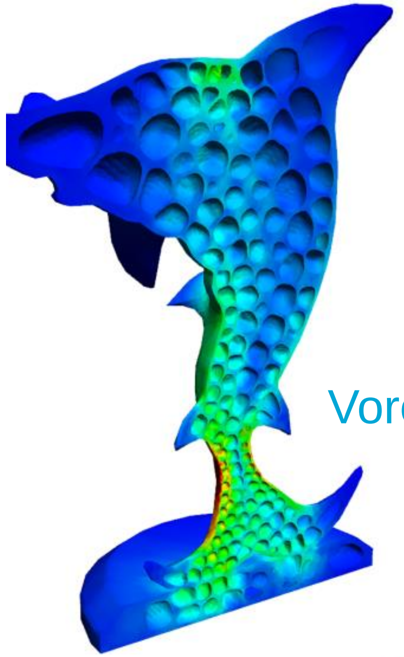
- Topology Optimization for Additive Manufacturing
  - Geometric feature control by **density filters**
  - Geometric feature control by **alternative parameterizations**

Geometric feature control by alternative parameterizations (An incomplete list)

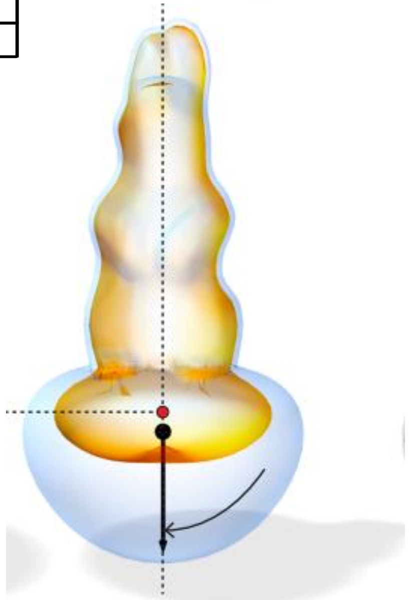


Reference: Voxel discretization

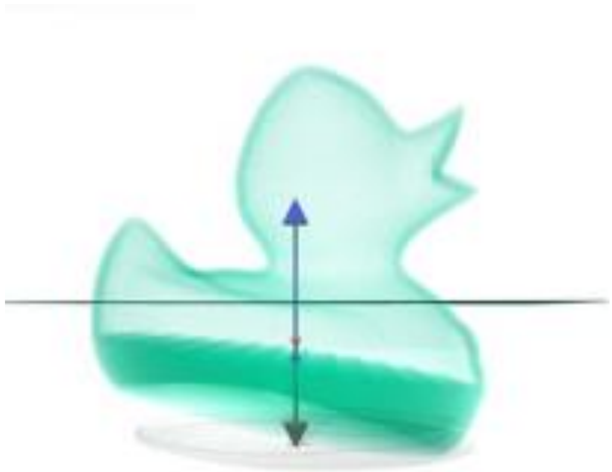
Skin-frame, Wang’13



Voronoi cells, Lu’14



Offset surfaces, Musialski’15

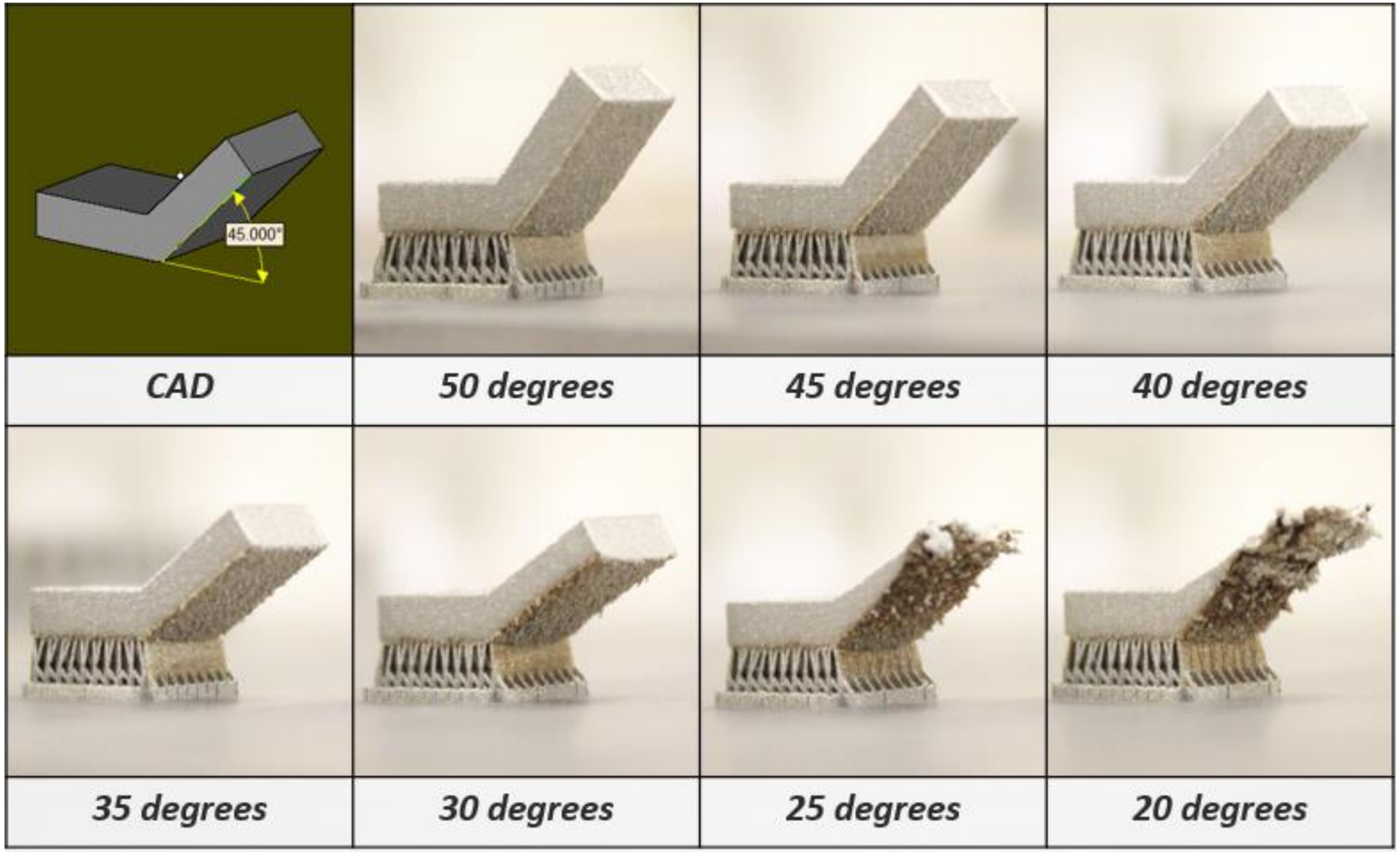


Ray representation, Wu’16

Adaptive rhombic, W

Overhang in Additive Manufacturing

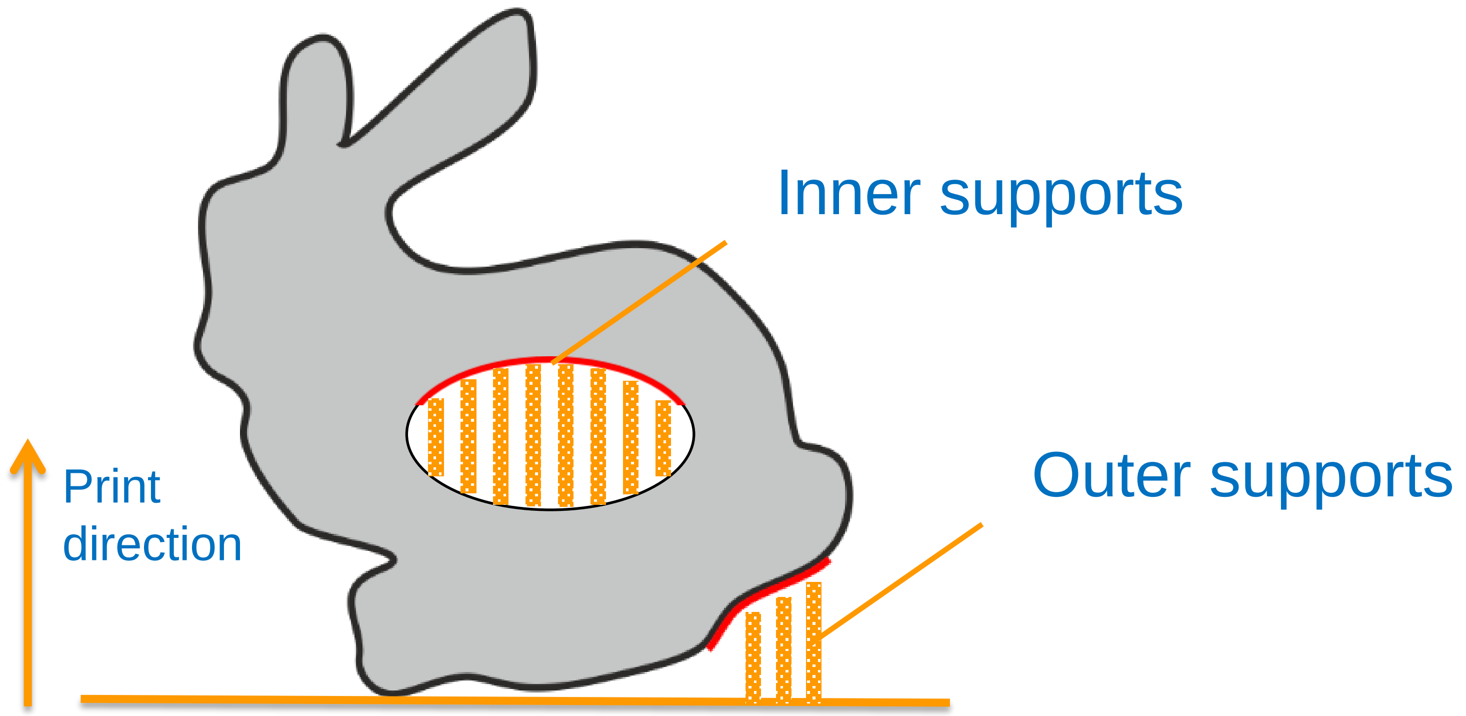
- Support structures are needed beneath overhang surfaces



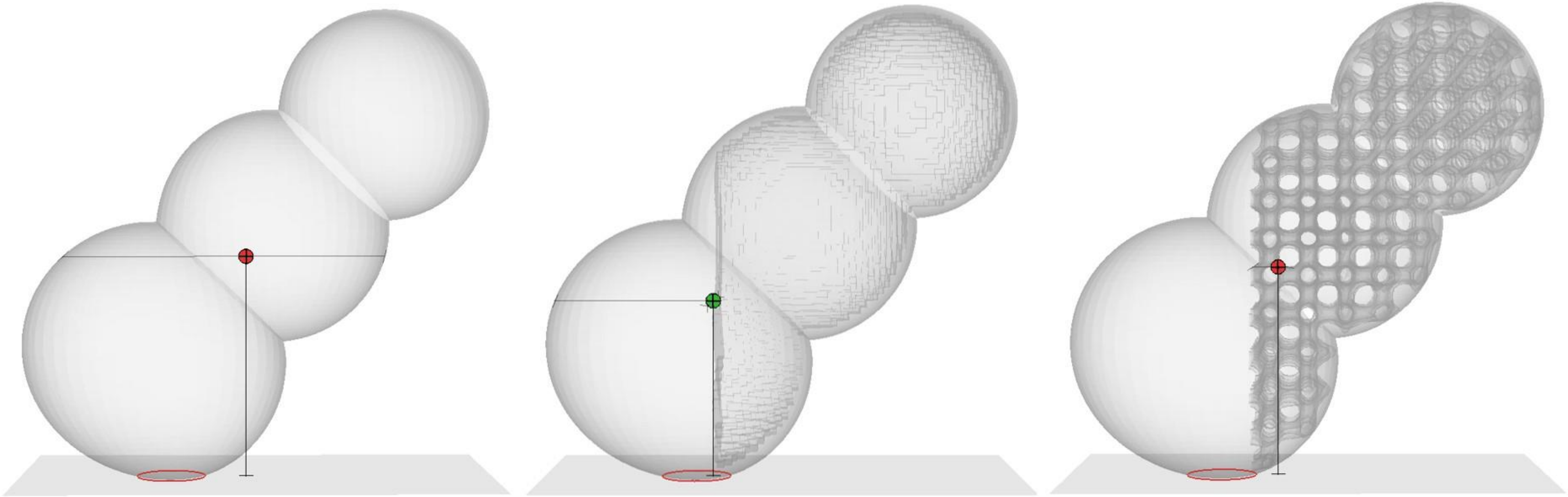
https://www.protolabs.com/blog/tag/direct-metal-laser-sintering/

Support Structures in Cavities

- Post-processing of inner supports is problematic



Infill & Optimization Shall Integrate



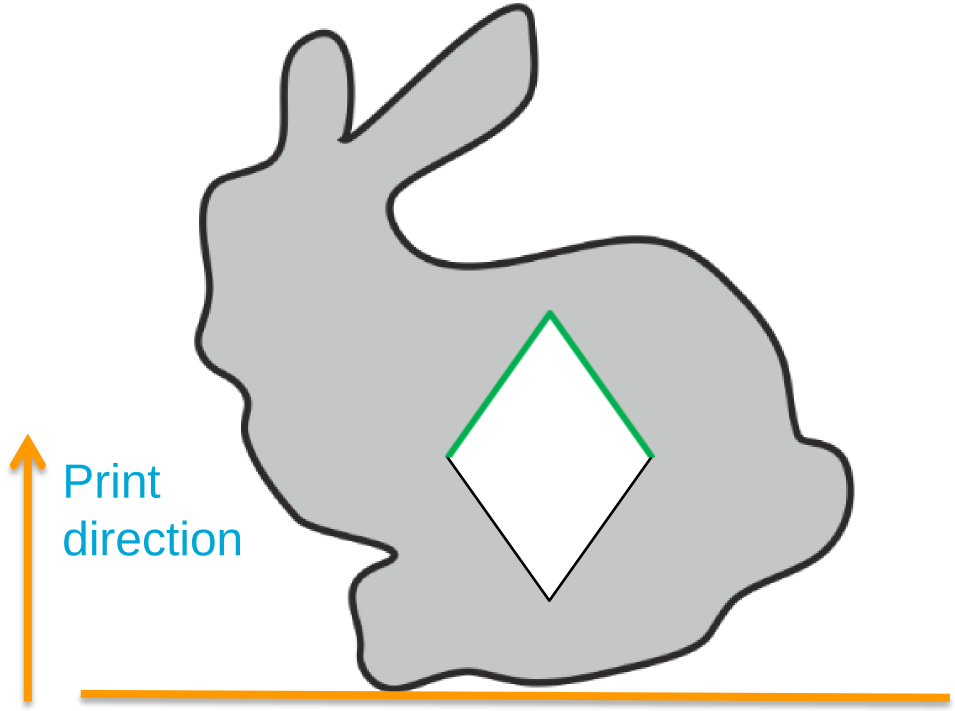
Solid,
Unbalanced

Optimized,
Balanced

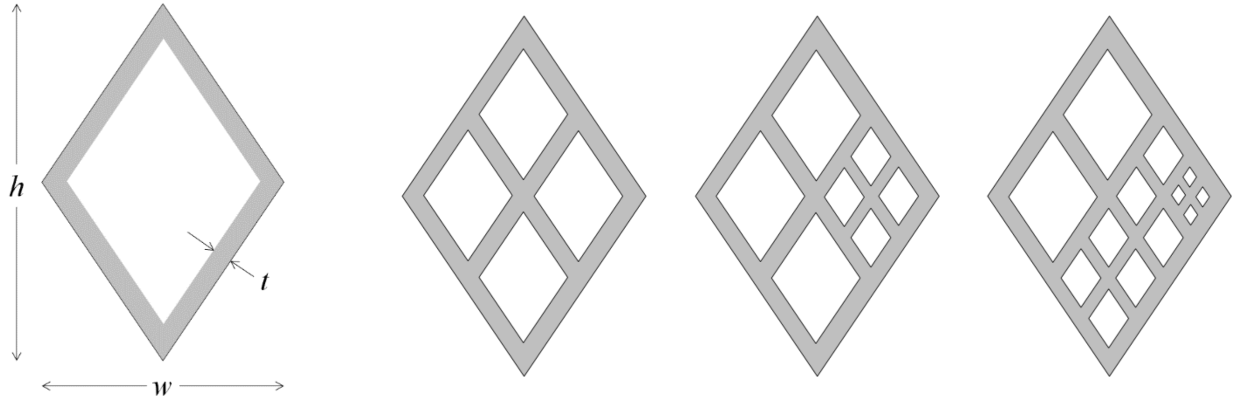
With infill,
Unbalanced

The Idea

- Rhombic cell: to ensure self-supporting
- Adaptive subdivision: as design variable in optimization

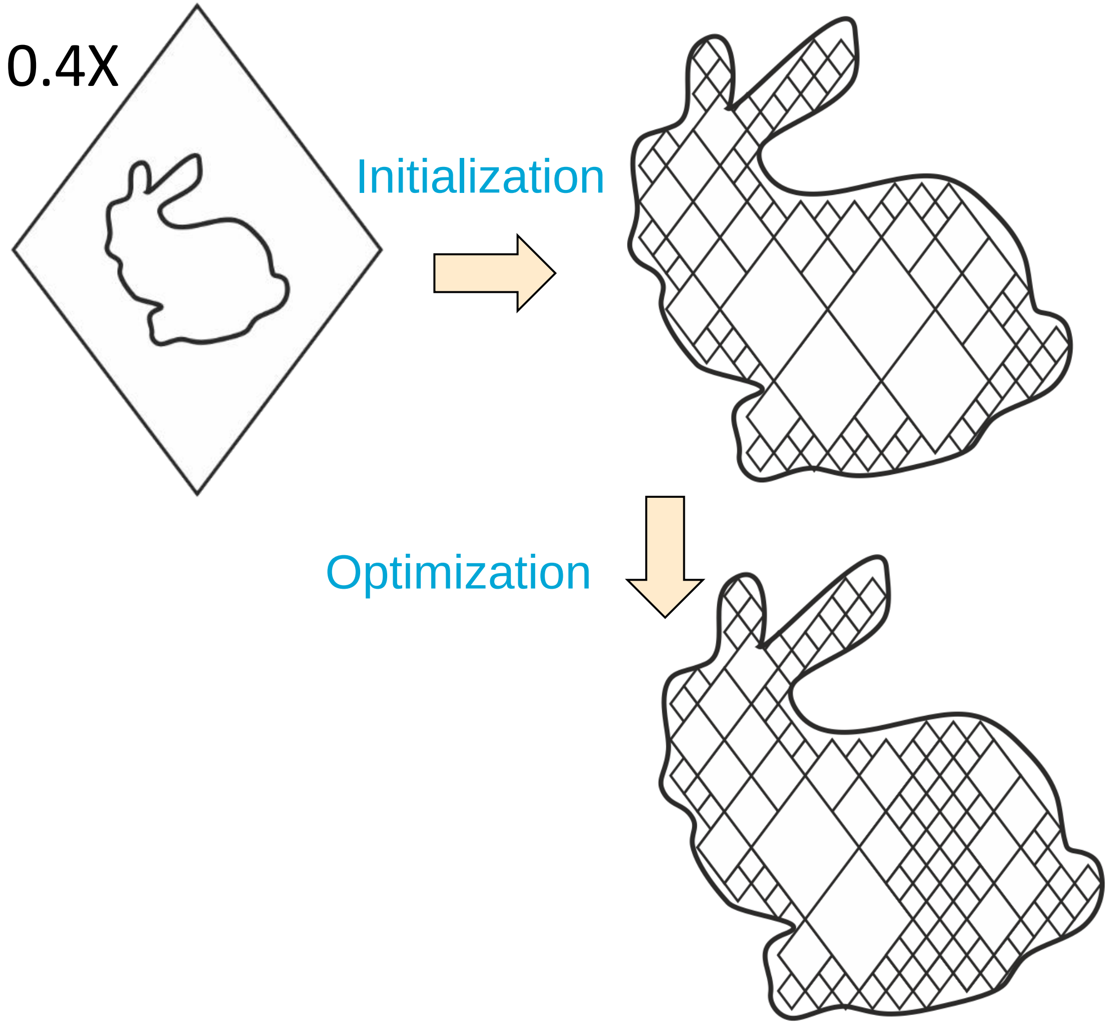


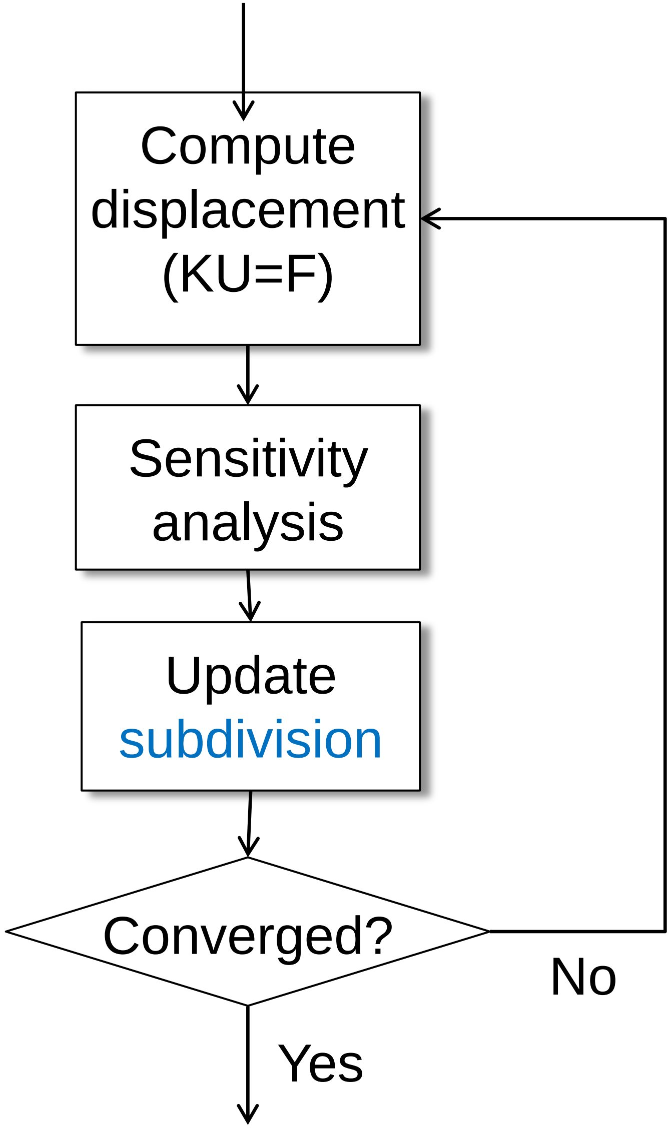
Rhombic cell



Adaptive subdivision

Self-Supporting Rhombic Infill: Workflow

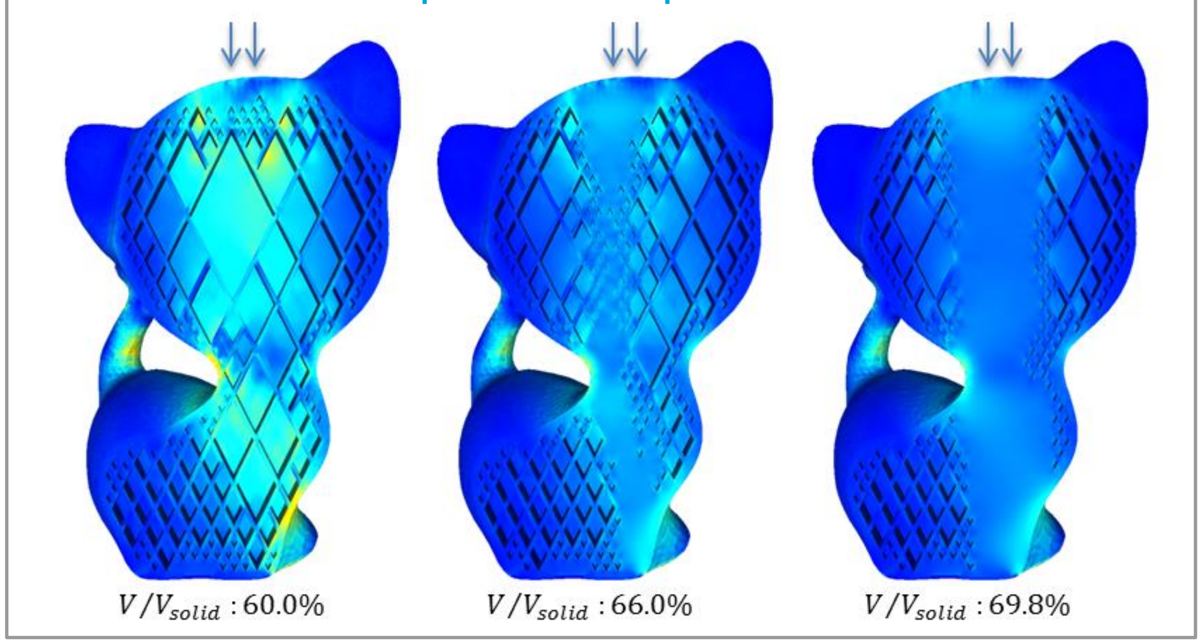




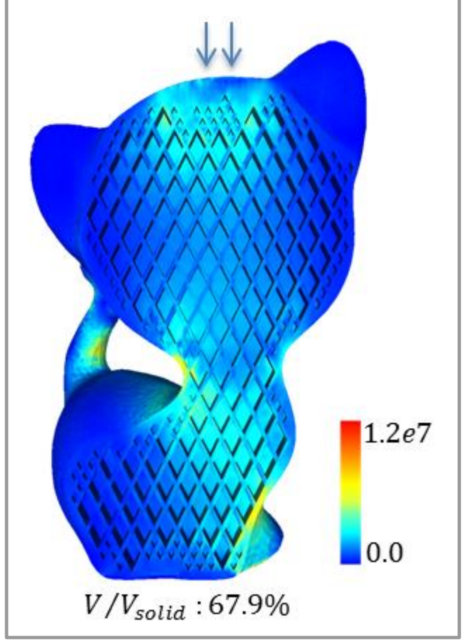
Self-Supporting Rhombic Infill: Results

- Optimized mechanical properties, compared to regular infill
- No additional inner supports needed

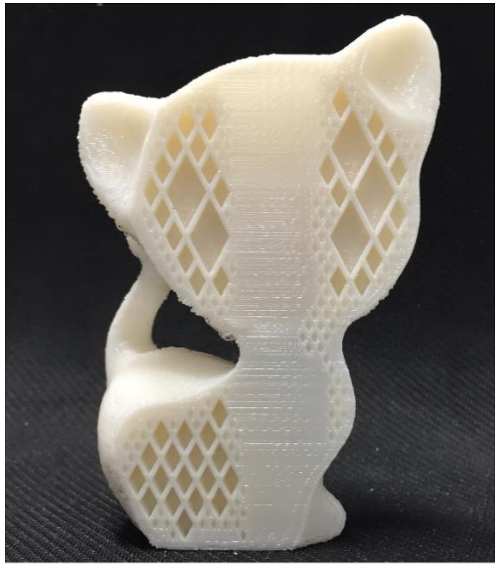
Optimization process



Reference

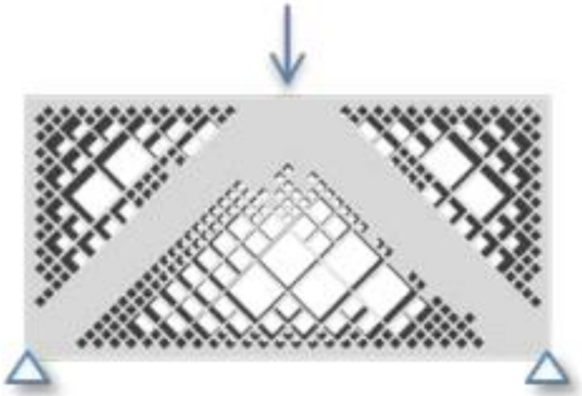


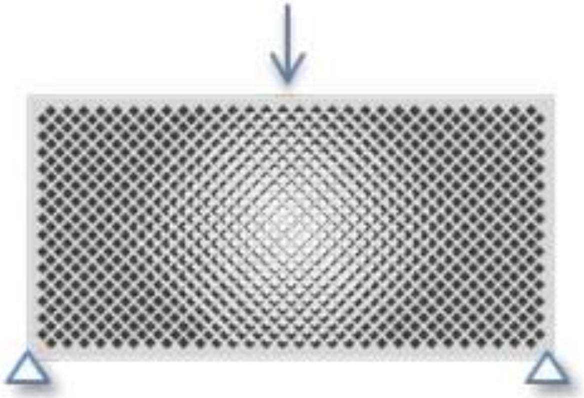
Print



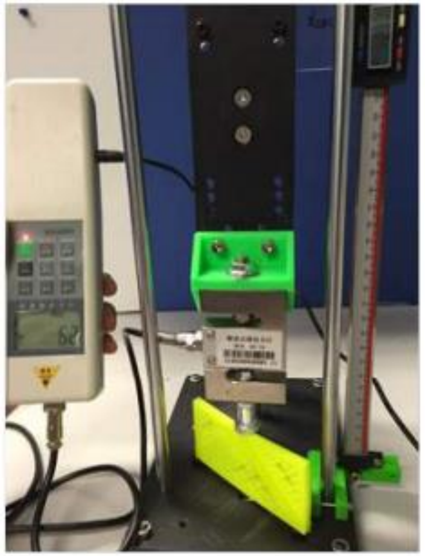
Wu et al., CAD'2016

Mechanical Tests





Under same force (62 N)



Dis.
2.11 mm

Dis.
4.08 mm

Under same displacement (3.0 mm)

Force
90 N

Force
58 N

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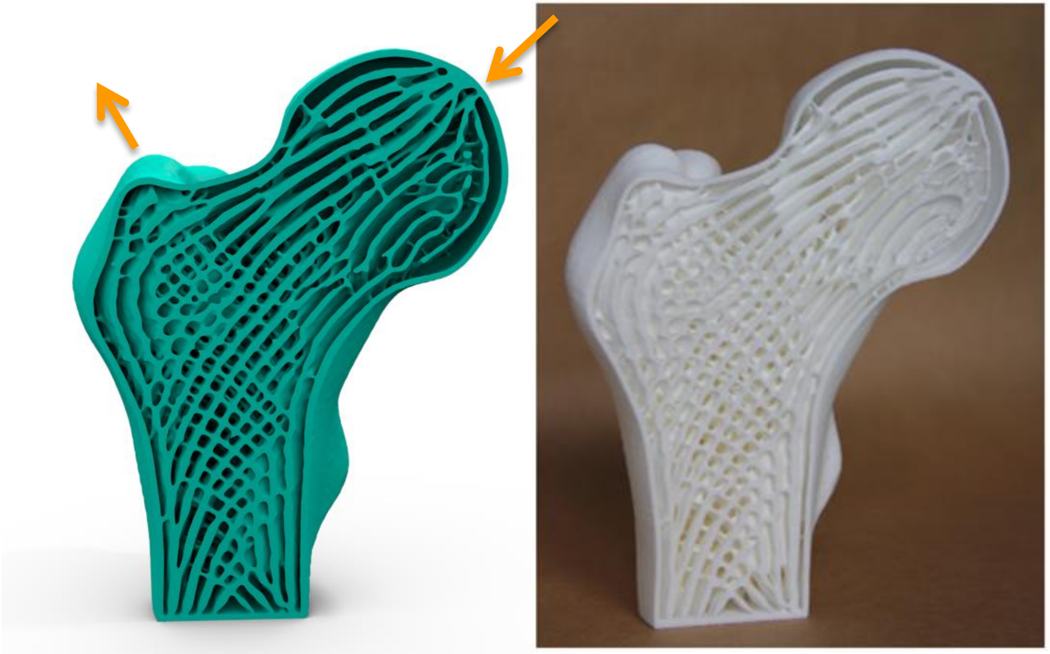
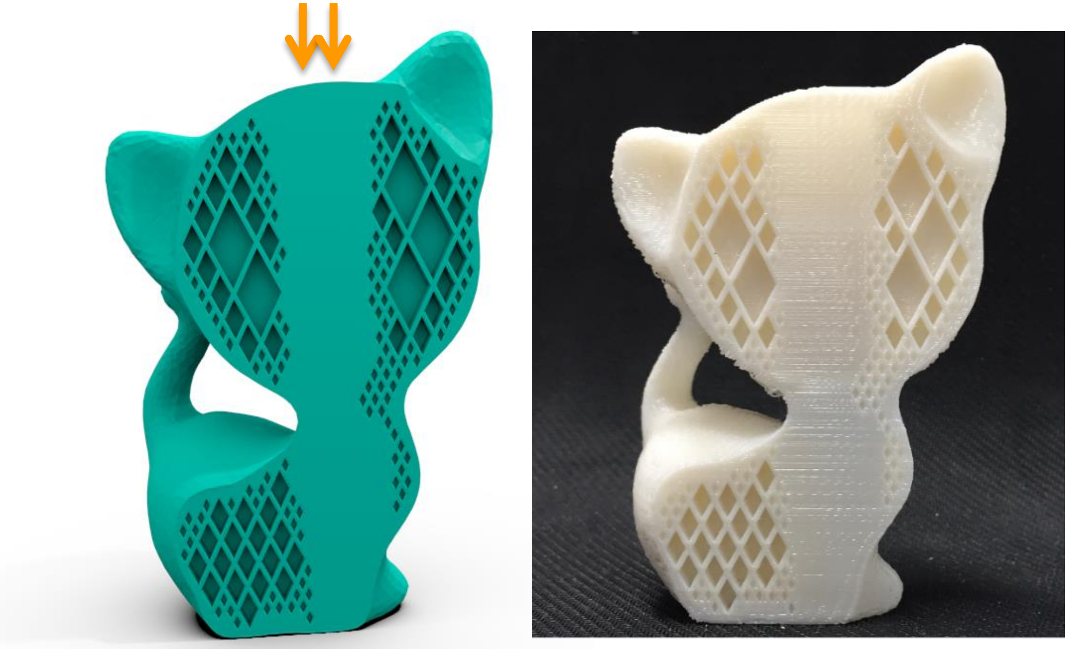


Bone-inspired infill



Self-supporting infill

Topology Optimization

- Lightweight
- Free-form shape
- Customization
- Mechanically optimized

Additive Manufacturing

- Customization
- Geometric complexity

Thank you for your attention!

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